Corporate Bond Yield Curves for Paraguay: A Reference Framework for Emerging Market Valuation

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Abstract

This study presents a methodology for constructing corporate bond yield curves in Paraguay's fixed-income market using the Nelson-Siegel-Svensson model with hierarchical credit rating constraints. Although yield curves for Paraguay's government instruments exist and international practice typically adds credit spreads for corporate valuation, spread estimation remains subjective because corporate bond markets lack a rigorous benchmark supported by economically consistent methodology. This study combines established techniques in a framework tailored to address this gap and delivers a systematic integration of: (i) an event-based cash flow solver that computes yields from actual corporate event schedules rather than approximation formulas; (ii) robust anchor construction using volume-weighted and maturity-weighted median yields across tenor buckets; and (iii) explicit correction of data artifacts to ensure that lower-rated bonds consistently price at higher yields than higher-rated bonds across all maturities—a fundamental credit market principle that unconstrained estimation often violates in data-sparse environments. The Nelson-Siegel-Svensson model's six-parameter structure captures level, slope, and curvature dynamics and accommodates the term structure patterns observed in emerging markets. Par yield curves are estimated for each currency-rating segment using Huber loss optimization with monotonicity penalties and isotonic regression smoothing. The resulting yield curves—both par yields for market benchmarking and zero-coupon yields for discounting applications— provide economically coherent references for securities valuation, risk assessment, and regulatory supervision and support both investment decisions and policy formulation in Paraguay's evolving fixed-income landscape.

Keywords: Yield Curve Estimation, Nelson-Siegel-Svensson Model, Corporate Bonds, Credit Rating Hierarchy, Par Yield Curve, Emerging Markets, Event-Based Pricing, Huber Loss, Optimization, Isotonic Regression, Brent's method .

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1 Introduction

Yield curves are fundamental tools in fixed-income analysis, providing a graphical representation of the relationship between bond yields and their maturities. They serve as benchmarks for pricing securities, assessing credit risk, managing portfolios, and formulating monetary policy. The Central Bank of Paraguay publishes a sovereign zero-coupon curve that is useful because international practice typically adds credit spreads for corporate valuation. However, without a standardized reference, spread estimation remains subjective and the corporate bond market lacks an established reference yield curve supported by a methodology that ensures economic consistency across credit rating categories.

This study develops a comprehensive framework for estimating Paraguay's corporate bond yield curves using the Nelson-Siegel-Svensson (NSS) model. The methodology addresses three critical challenges inherent to emerging market bond pricing: (i) accurate yield computation from complex bond structures with irregular cash flows; (ii) robust curve fitting in the presence of sparse and noisy transaction data; and (iii) correction of data artifacts to ensure that credit risk is properly reflected in yield differentials across rating categories.

The NSS model extends the original Nelson-Siegel framework by introducing an additional curvature factor, providing greater flexibility to capture the term structure patterns frequently observed in corporate bond markets. This six-parameter specification maintains the interpretability of the original model—with factors representing level, slope, and curvature—while accommodating more complex term structure dynamics.

While the NSS model provides the mathematical flexibility to fit complex term structures, a distinguishing feature of this study's implementation is the explicit correction of data artifacts that would otherwise produce economically inconsistent yield curves. In well-functioning markets, yields increase monotonically as credit quality deteriorates: AAA-rated bonds should offer lower yields than AA-rated bonds, which in turn should yield less than A-rated bonds, and so forth. However, in markets with limited liquidity and fragmented trading, unconstrained estimation can produce curves that violate this fundamental relationship because data sparsity, outliers, and idiosyncratic transactions introduce noise that distorts the estimated term structure. Our approach incorporates minimum spread constraints and hierarchical curve fitting to correct for these data artifacts, ensuring that the resulting yield curves reflect the underlying economic reality of credit risk pricing.

The framework employs an event-based cash flow solver that computes yields directly from actual corporate event schedules—including coupon payment dates, amortization schedules, and maturity terms—rather than relying on simplified approximation formulas. This approach produces more accurate yield estimates, particularly for bonds with non-standard payment structures common in the Paraguayan market.

This paper proceeds as follows: Section 2 reviews the relevant literature on yield curve modeling. Section 3 describes the data sources and provides a comprehensive analysis of market characteristics. Section 4 presents the model specification, including the NSS framework and credit hierarchy constraints. Section 5 details the estimation methodology. Section 6 presents the application results with in-depth analysis of the estimated yield curves. Section 7 addresses limitations and recommendations for future work. Section 8 presents alternative approaches that were explored. Section 9 concludes.

2 Literature Review

The estimation of yield curves has been a central theme in fixed-income research since the seminal work of Nelson & Siegel (1987), who proposed a parsimonious three-factor model capable of capturing the level, slope, and curvature of the term structure using exponentially decaying basis functions. This model gained widespread adoption due to its interpretability and ability to reproduce the typical shapes observed in government bond markets—upward-sloping, downward-sloping, flat, and humped curves.

Svensson (1994) extended the Nelson-Siegel framework by adding a second curvature term with an independent decay parameter, providing additional flexibility to fit more complex yield curve shapes. The Nelson-Siegel-Svensson (NSS) model has since become a standard tool employed by central banks worldwide, including the European Central Bank (Coroneo et al., 2008), the Bank for International Settlements (BIS, 2005), and numerous emerging market authorities.

However, the application of these models to corporate bond markets presents additional challenges not encountered in sovereign debt analysis. Elton et al. (2001) documented that corporate yield spreads reflect not only default risk but also tax effects and liquidity premia, suggesting that corporate yield curves exhibit fundamentally different dynamics than their government counterparts. Their work established empirical foundations for minimum credit spread relationships between rating categories—findings that inform the constraint calibration in this study.

In emerging markets, yield curve estimation faces particular difficulties due to limited liquidity, sparse transaction data, and market segmentation. Diebold & Li (2006) advocated for robust estimation techniques and cross-validation procedures to mitigate overfitting in data-constrained environments. Their analysis of medium-term tenors, particularly around the 5-year maturity, demonstrated that this segment best captures the "level" factor of the yield curve—a finding that motivates the anchor point selection in our methodology.

Gürkaynak et al. (2007) provided comprehensive guidance on parameter bounds and estimation procedures for Nelson-Siegel class models, establishing that coefficient values outside certain ranges produce economically implausible curve shapes. Their recommendations inform the optimization bounds that were employed in this study.

Gonçalves & Marques (2016) applied the Nelson-Siegel framework to Brazil's corporate bond market, demonstrating its applicability in Latin American contexts and highlighted the need for careful parameter calibration. Their work provided a regional precedent for the methodological approach adopted in this work.

While yield curves for government instruments are available for Paraguay (BCP, 2025c), no prior work has documented a rigorous methodology for constructing corporate bond yield curves that ensures economic consistency across credit rating categories. This study contributes to the literature by combining the NSS framework with explicit credit hierarchy constraints, robust estimation using Huber loss optimization, and event-based yield calculation.

3 Data Description and Market Analysis

3.1 Data Sources

The primary dataset comprises transaction-level records from the Bolsa de Valores de Asunción (BVA), Paraguay's sole stock exchange. The records include detailed information on trade execution, bond characteristics, pricing, and credit risk for all fixed-income transactions. Additionally, corporate event schedules—specifying coupon payment dates, amortization schedules, and maturity terms—are obtained from the exchange's securities registry.

The analysis covers a 36-month window from January 2023 through December 2025, encompassing a period of relative macroeconomic stability in Paraguay following the post-pandemic recovery. This time frame provides sufficient depth for robust estimation while maintaining relevance to current market conditions.

3.2 Market Overview

Table 1 presents summary statistics for Paraguay's fixed-income market during the analysis period. The market processed 61,557 transactions over 36 months, averaging approximately

1,710 transactions per month. Total trading volume reached USD 5.54 billion, with monthly volume averaging USD 153.9 million. These figures reflect a market of modest but sufficient size and activity to support yield curve estimation.

Table 1: Summary Statistics: Fixed-Income Market
Jan 2023 – Dec 2025

Statistic	**Value**
Analysis Period	36 months
Number of transactions	61,557
Avg. Transactions per Month	1,710
Total Volume Traded (USD)	$5,540,851,227
Avg. Volume Traded per Month (USD)	$153,912,534
Number of Unique Bond Issues	1,103
Number of Issuers	102
Number of Currencies	2
Number of Credit Rating Categories	7

The market features 1,103 unique bond issues from 102 distinct issuers, indicating reasonable diversity in the issuer base despite the concentration patterns discussed below. Trading occurs in two currencies—Paraguayan Guaraní (PYG) and U.S. Dollars (USD)—reflecting the dual-currency nature of Paraguay's financial system, where dollar-denominated instruments play a significant role due to historical dollarization patterns and investor preferences for currency diversification.

3.3 Market Characteristics

The Paraguayan securities market is dominated by fixed-income instruments, which account for approximately 97% of total trading volume. Credit ratings follow local conventions established by the Superintendency of Securities, with granular designations (e.g., AAA+, AAA, AAA-) standardized into broad categories for modeling purposes: investment grade (AAA, AA, A, BBB), speculative grade (BB, B) and non-performing or distressed (C, D, E) categories. However, only seven rating categories (AAA through C) were actively traded during the analysis period.[1]

The market exhibits characteristics typical of emerging economies: a concentrated issuer base, limited secondary market liquidity, and uneven trading activity across maturity segments. These features motivate the methodological choices described in subsequent sections, particularly the use of robust estimation techniques and explicit constraint enforcement.

3.3.1 Data Selection and Classification

For yield curve construction, the dataset is filtered according to the following criteria:

- **Instrument types:** corporate bonds (Bono), financial bonds (Bono Financiero), subordinated bonds (Bono Subordinado), and zero-coupon bonds (Bonos Bursatiles de Corto Plazo or BBCP).[2]

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[1] Crucially, although these categories see market activity, not all of them possess sufficient liquidity or a diverse enough set of tenors to meet the strict criteria required for the construction of a robust yield curve.

[2] The inclusion of zero-coupon instruments, despite their small market share (0.6% of transactions), is justified by their contribution to short-end coverage where coupon-bearing bonds may be sparse. See Appendix A.1.1 for detailed rationale and statistics.

- **Market segments:** Primary and secondary market transactions.[3]
- **Sectors:** all sectors except public administration issuers.

Residual maturity is classified into discrete tenor buckets at half-year intervals from 0.5 to 10 years. This bucketing serves to aggregate sparse observations into more robust yield estimates, and provide a standardized grid for curve interpolation. The tenor classification follows:

$$\tau_i = \begin{cases} 0.5 & \text{if } t \leq 0.5 \\ 1.0 & \text{if } 0.5 < t \leq 1.0 \\ \vdots & \\ 10.0 & \text{if } t > 9.5 \end{cases} \tag{1}$$

where t denotes the residual maturity in years and τ_i represents the assigned tenor bucket.

3.3.2 Trading Activity Patterns

Several dimensions of market activity inform the estimation methodology:

Issuer concentration. The top 10 issuers account for 60.4% of total trading volume, with financial institutions predominating the list. This concentration implies that certain rating-tenor segments may be influenced by issuer-specific effects, and motivates the robust estimation techniques employed in Section 5. Detailed issuer statistics are provided in Appendix A.1.2.

Primary vs. secondary market structure. The secondary market accounts for 79.5% of transactions but only 46.6% of volume, while the primary market captures 53.4% of volume with just 20.5% of transactions. This asymmetry reflects buy-and-hold behavior among institutional investors, with the average primary market transaction approximately 3.9 times larger than secondary market trades. The methodology accounts for this pattern by volume-weighting observations, which ensures that larger transactions receive appropriate influence in anchor point construction. Detailed market segment analysis is provided in Appendix A.1.3.

Maturity distribution. Trading activity concentrates heavily in the 1.5–5 year maturity segment, with the 3-year tenor showing highest activity (approximately 175 monthly transactions in PYG, 80 in USD). Longer maturities exhibit progressively sparser trading: the 8+ year segment averages fewer than 25 transactions monthly per currency, implying fewer than 6 transactions per week spread across multiple rating categories. This sparsity motivates the use of robust estimation techniques, maturity-weighted observation schemes, and the NSS model's smooth interpolation across tenor gaps. Complete maturity distribution statistics are provided in Appendix A.1.4.

Rating-tenor coverage. Investment-grade categories show adequate coverage across most tenor buckets, with peak density in the A-rated 3-year PYG segment (101 average monthly transactions). Speculative-grade ratings exhibit substantially sparser coverage, averaging fewer than 8 transactions per month across most tenor buckets. The C rating shows near-total illiquidity, making it unsuitable for term structure estimation. For USD, the AAA segment has limited coverage (fewer than 15 monthly transactions in most tenors), explaining the intermittent availability of the USD AAA curve. A complete rating-tenor heatmap is provided in Appendix A.1.5.

Coupon Payment Frequency The methodology estimates par yield curves assuming quarterly coupon payments ($f = 4$). In market practice, quarterly coupons account for 69.7% of all transactions and 63.6% of unique bond issues, representing the clear modal convention in Paraguay's corporate bond market. The complete distribution of payment frequencies is provided in Appendix A.1.6.

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[3] Repurchase agreements (repos) are excluded as they represent secured financing transactions rather than outright bond purchases. Repo pricing reflects money market conditions rather than term credit risk.

3.3.3 Instrument Types and Grouping Rationale

The dataset encompasses four bond instrument types: corporate bonds (Bono), financial bonds (Bono Financiero), subordinated bonds (Bono Subordinado), and zero-coupon bonds (BBCP). Here, cross-sector yield variation within the same rating category (up to 450 basis points for BBB-rated bonds) substantially exceeds instrument-type differences within sectors (typically below 150 basis points). This suggests that credit ratings capture the primary systematic risk factor, while instrument-type distinctions primarily reflect issuer sector composition.

Given data constraints—many sector-rating combinations average fewer than 5 monthly transactions—the methodology groups all coupon-bearing instrument types together, relying on credit ratings as the primary risk stratification. As market depth increases, future refinements may enable sector-differentiated estimation. Detailed sector-by-instrument yield comparisons are provided in Appendix A.1.7.

4 Model Specification

4.1 Nelson-Siegel-Svensson Framework

The Nelson-Siegel-Svensson (NSS) model represents the zero-coupon yield curve as a function of maturity using six parameters. The model extends the original three-parameter Nelson-Siegel specification by adding a second curvature term, providing additional flexibility to capture complex yield curve shapes including double-humped patterns.

The NSS model specifies the continuously compounded zero-coupon (spot) rate at maturity τ as:

$$r(\tau;\boldsymbol{\theta}) = \beta_0 + \beta_1\left(\frac{1-e^{-\tau/\lambda_1}}{\tau/\lambda_1}\right) + \beta_2\left(\frac{1-e^{-\tau/\lambda_1}}{\tau/\lambda_1} - e^{-\tau/\lambda_1}\right) + \beta_3\left(\frac{1-e^{-\tau/\lambda_2}}{\tau/\lambda_2} - e^{-\tau/\lambda_2}\right) \tag{2}$$

where $\boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2)^\top$ is the parameter vector.

Each parameter $\boldsymbol{\theta}$ has a specific economic interpretation that aids in understanding curve behavior:

- β_0 **(Level):** Controls the overall level of the yield curve. As maturity approaches infinity, the yield converges to β_0. This parameter represents the long-term equilibrium interest rate and shifts the entire curve up or down. When β_0 increases, all yields across the maturity spectrum rise proportionally.

- β_1 **(Slope):** Determines the slope of the yield curve, particularly at shorter maturities. A negative β_1 produces an upward-sloping curve (normal), while a positive β_1 produces a downward-sloping curve (inverted). The sum $\beta_0 + \beta_1$ equals the instantaneous short rate—the theoretical yield at maturity zero. Thus β_1 captures the spread between short-term and long-term rates.

- β_2 **(First Curvature):** Creates a hump or trough in the yield curve at medium-term maturities. The location of this hump is governed by λ_1. A positive β_2 creates a hump (yields rise then fall), while negative β_2 creates a trough. This term allows the model to capture the "belly" effects often observed in corporate credit curves.

- β_3 **(Second Curvature):** Adds a second hump or trough at a different maturity, governed by λ_2. This additional flexibility allows the model to fit more complex yield curve shapes, such as double-humped patterns sometimes observed in corporate credit curves when different market segments exhibit distinct dynamics.

- **λ_1, λ_2 (Decay Parameters):** Control the speed at which the exponential terms decay and thus where the curvature effects are maximized. Smaller values concentrate effects at shorter maturities; larger values spread effects to longer maturities. These parameters effectively "tune" the maturity at which the humps or troughs occur.

4.2 Par Yields

Given the NSS zero-coupon yield curve, the discount function is:

$$D(\tau) = \exp\left(-r(\tau; \boldsymbol{\theta}) \cdot \tau\right) \tag{3}$$

The discount function used here represents the present value of a unit payment (e.g., 1 PYG or 1 USD) to be received at time τ, derived from the continuously compounded annualized zero-coupon yield $r(\tau)$. This exponential form is the cornerstone of term structure modeling, as it ensures a consistent and arbitrage-free link between the yield curve and the price of risk-free obligations across different maturities.

The par yield $c(\tau)$ is the coupon rate at which a bond of maturity τ with coupon frequency f prices at par:

$$c(\tau) = \frac{1 - D(\tau)}{\sum_{i=1}^{n} \Delta_i \cdot D(t_i)} \tag{4}$$

where t_i denotes the coupon payment dates, Δ_i is the day count fraction for period i, and $n = f \cdot \tau$ is the total number of coupon periods.

Economically, the par yield $c(\tau)$ defined in Equation 4 is the internal rate of return that equates the present value of a bond's future cash flows—both periodic coupons and the final principal repayment—to its current face value. In Equation 4, the numerator $(1 - D(\tau))$ captures the present value of the "loss" or discount on the principal repayment due to the time value of money. The denominator acts as an annuity factor (or "DV01"), representing the sum of the present values of a sequence of unit coupon payments over the life of the bond. Setting the par yield in this manner effectively finds the equilibrium coupon rate where the interest earned by the investor exactly offsets the erosion of the principal's value over time. In the Paraguayan context, this calculation assumes quarterly payments ($f = 4$), mirroring the liquidity and cash-flow patterns observed in local secondary market transactions documented in Section 3.

It is important to note that the NSS model fundamentally produces zero-coupon yields through Equation 2, from which discount factors are derived via Equation 4. Par yields, as shown in Equation 4, are then computed as a function of these discount factors. This sequential relationship—zero rates first, par yields second—reflects the theoretical foundation of term structure modeling, where spot rates represent the primitive discount function from which all other yield measures are derived. The estimation methodology (Section 5) fits the NSS parameters by minimizing deviations between model-implied par yields and observed market yields, but the fitted parameters simultaneously define both the zero-coupon and par yield curves. Both curve types serve as reference rates, providing distinct but complementary purposes in fixed-income analysis.

4.3 Credit Hierarchy Constraints

A fundamental principle of credit markets is that lower-rated bonds must offer higher yields to compensate investors for increased default risk. Formally, for any maturity τ the yields y should follow:

$$\text{For yields } y \in \{r, c\}: \quad y_{AAA}(\tau) < y_{AA}(\tau) < \cdots < y_B(\tau) \tag{5}$$

In markets with sparse data, unconstrained curve estimation may violate this hierarchy due to sampling noise, outliers, or unrepresentative transactions. The methodology corrects for these data artifacts through two mechanisms:

Minimum Spread Constraints: Each rating category must maintain a minimum spread over the AAA curve. These spreads are calibrated based on international empirical evidence. Elton et al. (2001) documented corporate spread relationships in U.S. markets, finding that credit spreads increase systematically with lower ratings even after controlling for expected default losses. According to Moody's (2024), cumulative five-year default rates for AAA-rated issuers average below 0.5%, while B-rated issuers experience cumulative default rates exceeding 20% over the same horizon.

Table 2: Minimum Spread Constraints Over AAA

Rating	**Minimum Spread (bps)**
AAA	0 (base)
AA	50
A	100
BBB	150
BB	200
B	250

Sequential Hierarchy Enforcement: Curves are fitted in credit quality order (AAA first, then AA, etc.), with each curve constrained to maintain at least a minimum spread over AAA at every tenor point.

5 Estimation Methodology

This section details the estimation procedure for constructing corporate bond yield curves. The methodology proceeds in four stages: (i) computing transaction-level yields from observed prices, (ii) constructing robust anchor points at each tenor, (iii) fitting the NSS model with penalized optimization, and (iv) enforcing credit hierarchy constraints across rating categories. To facilitate reading, a complete summary of the mathematical notation employed throughout this paper is provided in Appendix A.2.

5.1 Event-Based Yield Calculation

A key innovation of this work is the computation of yields from actual corporate event schedules rather than simplified approximation formulas. The yield-to-maturity (YTM) is computed for each transaction to provide comparable yield observations across bonds with different coupon rates, payment frequencies, and maturities by solving for the internal rate of return that equates the dirty price to the present value of all future cash flows.

5.1.1 Price and Cash Flow Concepts

Bond prices in the market are typically quoted as clean prices per 100 units of outstanding principal. However, yield calculations require the dirty price (also called the invoice price), which is what the buyer actually pays. The dirty price equals the clean price plus accrued interest—the portion of the next coupon payment that has accumulated since the last payment date.

For amortizing bonds where some principal has already been repaid, the outstanding principal at any point is less than the original face value. The methodology tracks outstanding principal based on the actual amortization schedule from corporate events, ensuring accurate price scaling.

Corporate events are extracted from the BVA registry and include:

- Interest payments (coupon dates and amounts)
- Amortization payments (principal redemptions)
- Maturity redemption

For each bond, the cash flow schedule is constructed as:

$$CF_t = I_t + A_t \tag{6}$$

where CF_t is the total cash flow at time t, I_t is the interest payment, and A_t is the amortization payment (including final principal redemption).

5.1.2 Yield Solver

For each observed transaction, the effective annual yield $\tilde{y}$ is obtained by solving the pricing equation:

$$P_{dirty} = \sum_{i:t_i > t_s} \frac{CF_{t_i}}{(1+\tilde{y})^{(t_i - t_s)/365}} \tag{7}$$

where CF_{t_i} represents the cash flow at time t_i, t_s is the settlement date, and P_{dirty} is the dirty price per 100 units of original face value. The tilde notation ($\tilde{y}$) distinguishes transaction-level observed yields from the model-implied curve rates.

The nominal yield-to-maturity $y_i^{(f)}$ is then derived from $\tilde{y}$ based on the coupon frequency f:

$$y_i^{(f)} = f \cdot \left[(1+\tilde{y})^{1/f} - 1\right] \tag{8}$$

For bonds with annual coupons ($f = 1$), $y_i = \tilde{y}$. The solver employs root-finding via Brent's method with dynamically expanding bounds to ensure convergence across a wide range of price-yield combinations.

The transaction-level (y_i), serve as inputs for anchor point construction. The full mathematical specification is provided in Appendix A.3.

5.2 Anchor Point Construction

The transaction-level y_i values computed in Section 5.1.2 serve as the raw inputs for curve estimation. Rather than fitting the NSS model directly to individual transaction yields, the methodology first constructs robust anchor points at each tenor bucket. This approach reduces the influence of outliers and provides stable targets for curve fitting.

Anchor points are only constructed for tenor buckets with at least two observations. While many practitioners use higher thresholds (5 or 10 observations), this lower threshold is a practical constraint necessary for the Paraguayan market. As documented in Section 3 (Figure 8), certain rating-tenor-currency combinations have very limited observations, particularly for speculative-grade bonds and longer maturities. A higher threshold would exclude too many tenor buckets, leaving insufficient anchor points for meaningful curve estimation. Buckets with insufficient data are excluded from the fitting process, with the NSS model interpolating yields at those tenors.

5.2.1 Winsorization

Before computing anchor yields, the transaction-level y_i values are winsorized to the 10th–90th percentile range within each tenor bucket. Winsorization is a statistical technique that limits the influence of extreme values by replacing them with less extreme values at specified percentiles.

$$y_i^w = \max\left(q_{10}, \min\left(q_{90}, y_i\right)\right) \tag{9}$$

where y_i^w is the winsorized YTM, y_i denotes the original YTM for transaction i, and q_{10}, q_{90} are the 10th and 90th percentiles of YTM within that bucket.

Unlike trimming, which removes outliers entirely, winsorization caps them—pulling extreme high values down to the 90th percentile and extreme low values up to the 10th percentile. This preserves the sample size while reducing the impact of potentially erroneous or unrepresentative transactions.

5.2.2 Weighting Scheme

For each combination of currency, credit rating, and tenor bucket, the anchor yield is computed as a weighted median of the winsorized observations. This weighting scheme reflects the principle that larger, more liquid transactions provide more reliable price signals, while rarer long-tenor observations receive additional weight to ensure adequate representation across the maturity spectrum.

Transaction weights combine volume and maturity considerations:

$$w_i = w_i^{vol} \cdot w_i^{mat} \tag{10}$$

The volume weight reflects trading activity:

$$w_i^{vol} = \log(1 + V_i) \tag{11}$$

where V_i is the transaction volume for transaction i. The logarithm serves an important purpose: it prevents a few extremely large transactions from completely dominating the estimate while still giving meaningful additional weight to larger trades.[4]

The maturity weight up-weights longer-tenor observations, which are typically scarcer, ensuring adequate representation across the maturity spectrum.

$$w_i^{mat} = \min\left(1, \max\left(0.5, \tau_i\right)\right) \tag{12}$$

where τ_i is the residual maturity (in years) of transaction i. Maturities below 0.5 years receive weight 0.5; maturities above 1 year receive weight 1.

5.2.3 Weighted Median Yields

For each combination of currency, credit rating, and tenor bucket, the anchor yield is computed as a weighted median:

$$\bar{y}_\tau = \text{wmedian}\left(\{y_i^w\}_{i \in B_\tau}, \{w_i\}_{i \in B_\tau}\right) \tag{13}$$

where $\bar{y}_\tau$ is the anchor yield at tenor τ, B_τ denotes the set of transactions in tenor bucket τ, y_i^w is the winsorized yield for transaction i, and w_i is the associated weight.

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[4] Without the log transformation, a single trade 100 times larger than average would have 100 times more influence; with the log transformation, its influence is proportionally reduced.

5.3 NSS Parameter Optimization

The NSS parameters $\boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2)$ are estimated by minimizing a penalized objective function that balances fit accuracy, curve smoothness, and economic consistency.

The complete objective function combines Huber loss with penalty terms:

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{\tau} w_\tau \cdot L_H\left(\bar{y}_\tau - \hat{c}(\tau; \boldsymbol{\theta})\right) + \alpha_1 P_{mono} + \alpha_2 P_{level} + \alpha_3 P_{band} \tag{14}$$

where $\bar{y}_\tau$ is the anchor yield at tenor τ, $\hat{c}(\tau; \boldsymbol{\theta})$ is the model-implied par yield, $L_H(\cdot)$ is the Huber loss function (defined below), $w_\tau = 1 + 2(\tau/\tau_{max})$ up-weights longer tenors with τ_{max} being the maximum tenor in the grid, and $\alpha_1, \alpha_2, \alpha_3$ are penalty weights calibrated to balance fit accuracy against regularization. The penalty terms P_{mono}, P_{level}, and P_{band} are defined in the following subsections.

Huber Loss. The primary fitting criterion employs Huber loss (Huber, 1964), which behaves quadratically for small residuals but transitions to linear growth for large residuals, limiting the influence of outliers common in emerging market transaction data. This approach follows central bank practice, including the Bank of Canada (Bolder & Gusba, 2002) and the Bank of England (Anderson & Sleath, 2001).

Monotonicity Penalty (P_{mono}). A penalty term discourages non-monotonic par yield curves, reflecting the economic rationale that investors require term premia for bearing duration risk over longer horizons (Diebold & Li, 2006).

Level Anchor (P_{level}). Following Diebold & Li (2006), who demonstrated that medium-term tenors best capture the yield curve's "level" factor, a penalty centers the curve near observed market yields at the 5-year tenor.

Band Constraint (P_{band}). An additional penalty prevents the fitted curve from extrapolating beyond observed yield ranges, mitigating against overfitting.

Parameters are estimated using the L-BFGS-B algorithm with box constraints (Diebold & Li, 2006; Gürkaynak et al., 2007). Complete mathematical specifications of penalty functions, calibrated weights, and parameter bounds are provided in Appendix A.4.

5.4 Post-Estimation Refinement and Curve Construction

The fitted NSS parameters $\hat{\boldsymbol{\theta}}$ obtained from the optimization in Section 5.3 produce both zero-coupon and par yields evaluated on a standardized grid from 0.5 to 10 years at quarterly intervals. However, these two outputs receive different post-processing treatments to ensure practical usability.

5.4.1 Zero-Coupon Yields: Direct NSS Output

Zero-coupon yields are computed directly from the fitted parameters using Equation 2. These zero yields preserve the exact functional form of the NSS model without further smoothing. This approach maintains theoretical consistency: zero rates represent the foundation of the term structure, and any additional smoothing would distort the internally consistent relationship between spot rates, forward rates, and discount factors inherent in the NSS framework. The discount factors derived from these zero rates satisfy the no-arbitrage condition by construction.

5.4.2 Par Yields: Isotonic Regression Smoothing

To ensure strictly non-decreasing term structures, we apply isotonic regression to the fitted par yields as a post-optimization refinement. This non-parametric technique finds the monotonic sequence $\hat{y}^{iso}$ that minimizes squared deviations from the raw fitted yields:

$$\hat{c}^{iso}(\tau_1) \leq \hat{c}^{iso}(\tau_2) \leq \cdots \leq \hat{c}^{iso}(\tau_K) \tag{15}$$

where $\hat{c}^{iso}(\tau_j)$ denotes the isotonically smoothed par yield at tenor τ_j, and K is the number of grid points.

Following the framework of Barlow et al. (1972), the algorithm resolves local monotonicity violations—often caused by optimization noise or tight credit constraints—by pooling adjacent points into their weighted averages. This process "irons out" inconsistencies while preserving the overall level and shape of the curve, ensuring that the final par yields are weakly increasing in tenor—a necessary property for stable market pricing.

5.5 Hierarchical Curve Fitting

Curves are fitted sequentially in credit quality order to enforce the hierarchy constraint:

Algorithm 1 Hierarchical Yield Curve Fitting

```
Input: Transaction YTMs $\{y_i\}$ by rating, minimum spreads $\{s_k\}$
Output: Spot rate curves $\{\hat{r}_k(\tau)\}$ and par yield curves $\{\hat{c}_k(\tau)\}$ for each rating $k$
Construct anchors: $\bar{y}_{AAA}(\tau) \leftarrow \text{WeightedMedian}(\{y_i^w\}_{AAA})$
Fit AAA curve: $\hat{r}_{AAA}(\tau) \leftarrow \text{FitNSS}(\bar{y}_{AAA})$
base_curve ← $\hat{r}_{AAA}$
prev_curve ← $\hat{r}_{AAA}$

for $k \in \{AA, A, BBB, BB, B\}$ do
    Construct anchors: $\bar{y}_k(\tau) \leftarrow \text{WeightedMedian}(\{y_i^w\}_k)$
    for each anchor $\bar{y}_k(\tau)$ do
        $\bar{y}_k(\tau) \leftarrow \max(\bar{y}_k(\tau), \text{base\_curve}(\tau) + s_k)$        ▷ Floor to AAA + spread
    end for
    Fit curve: $\hat{r}_k(\tau) \leftarrow \text{FitNSS}(\text{adjusted anchors})$
    Apply isotonic regression
    Enforce floor: $\hat{r}_k(\tau) \leftarrow \max(\hat{r}_k(\tau), \text{prev\_curve}(\tau))$
    prev_curve ← $\hat{r}_k$
end for

Derive par yields: $\hat{c}_k(\tau) \leftarrow \text{ComputeParYield}(\hat{r}_k)$ for each $k$
```

The algorithm enforces two complementary constraints: (i) each curve must exceed the AAA base curve by at least the minimum spread specified in Table 2, and (ii) each curve must exceed the immediately preceding curve by at least 50 basis points. The binding constraint at any tenor depends on how the curves are spaced—if market data already produces adequate separation, the constraints do not bind; if data artifacts would produce hierarchy violations, the constraints correct them.

6 Results and Analysis

This section presents the estimated zero-coupon and par yield curves for the Paraguayan corporate bond market in December 2025. The NSS model yields zero-coupon (spot) rates as the primary output (Equation 2), from which par yields are derived assuming quarterly coupons (Equation 4). Zero rates provide the pure discount function for single cash flows, while par yields offer intuitive benchmarks for new coupon-bearing issuances.

6.1 Zero-Coupon Yield Curves

Zero-coupon yields represent the annualized return on hypothetical zero-coupon bonds, serving as the foundational discount rates for valuation and risk analysis (Hull, 2018; Tuckman & Serrat, 2011). Figure 1 shows the estimated curves by currency and credit rating.

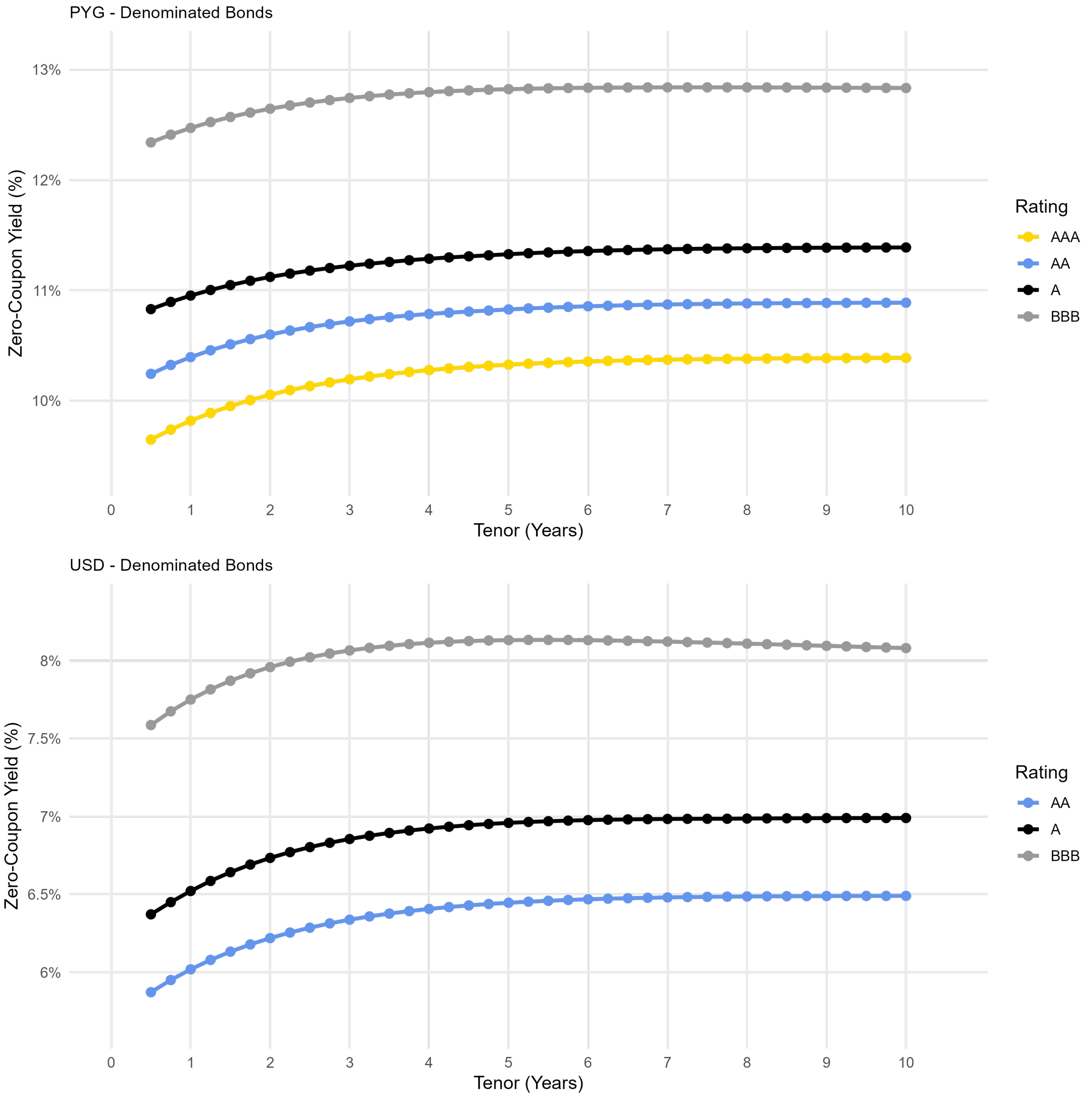

Figure 1: Zero-Coupon Yield Curves (December 2025).

PYG-denominated zero curves exhibit a modestly upward-sloping profile, with the AAA segment rising from 9.65% (6 months) to 10.39% (10 years), a 74 bps increase. Lower ratings follow similar patterns but at higher levels: AA (10.24%–10.86%), A (10.83%–11.37%), and BBB (12.34%–12.83%). Credit spreads over AAA are widest at shorter tenors (e.g., 265 bps for BBB at 1 year) and compress modestly at the long end (244 bps at 10 years), suggesting front-loaded credit premia. All PYG curves steepen sharply in the 0.5–2 year segment before flattening substantially beyond 3 years, consistent with expectations of stable long-term inflation and credit conditions.

USD-denominated zero curves lie 430–470 bps below their PYG counterparts at mid-to-long tenors (e.g., 5-year AA: 6.45% USD vs. 10.82% PYG), reflecting currency risk premia and expected depreciation. Credit spreads are materially tighter in USD (e.g., BBB–AA $\approx$ 168 bps at 5 years vs. 200 bps in PYG), attributable to the more homogeneous and higher-quality USD issuer pool (primarily financials and exporters). USD curves are also flatter, with total term rises of only 49–62 bps over 10 years, which may reflect a combination of factors including the concentrated issuer pool, limited long-term USD issuance, and potential correlation with U.S. Treasury term structure dynamics.

6.2 Par Yield Curves

Par yields indicate the coupon rate at which a newly issued quarterly-coupon bond would trade at par value, providing a practical reference for issuers and investors (Figure 2).

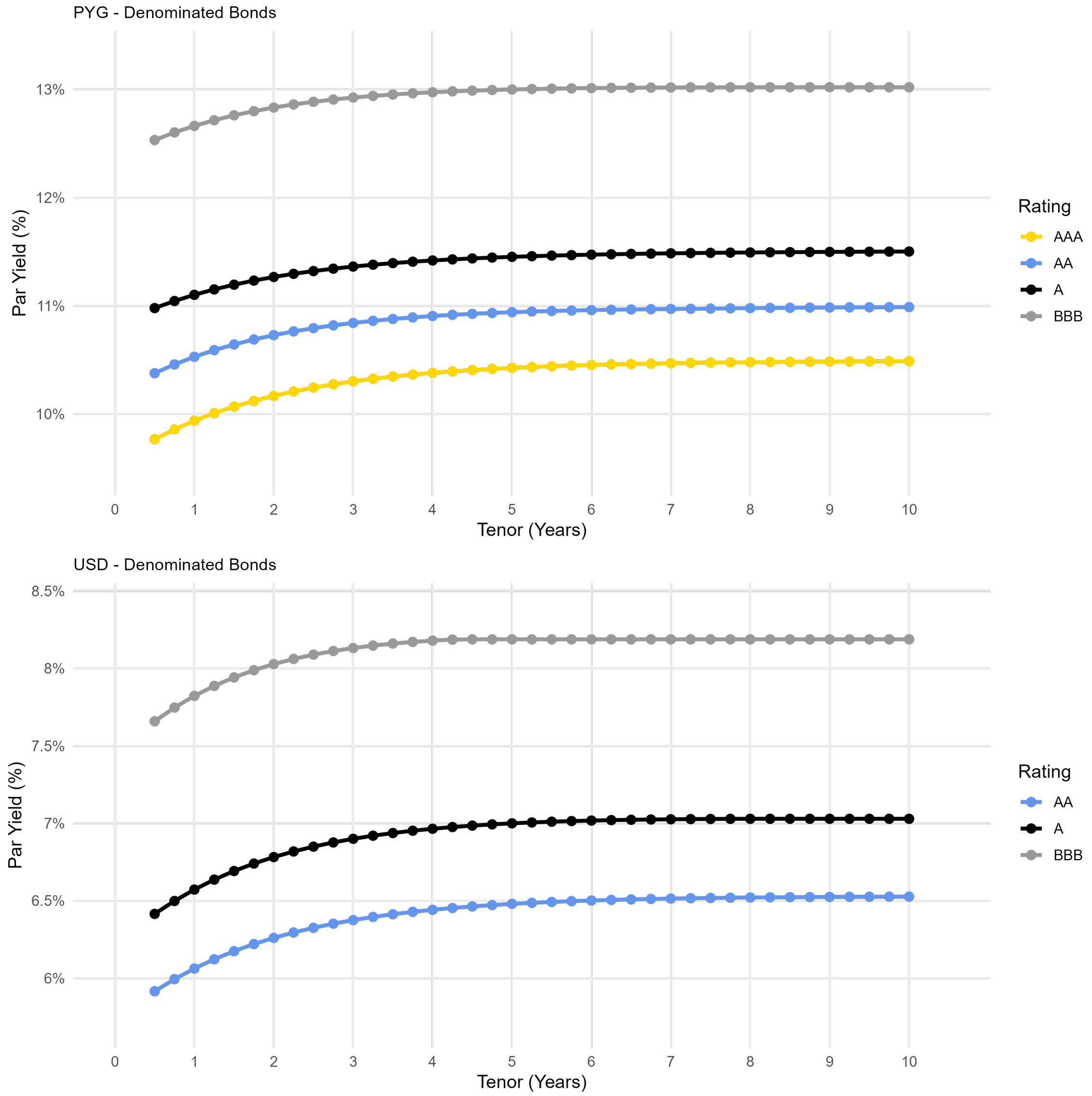

Figure 2: Par Yield Curves by Credit Rating (December 2025).

For PYG bonds, par curves closely track their zero-coupon counterparts, with AAA ranging from 9.76% (6 months) to 10.49% (10 years) and similar modest upward slopes for lower ratings. Par spreads are marginally wider than zero spreads (typically 1–9 bps), a small but systematic effect arising from quarterly coupon reinvestment in an upward-sloping environment. The front-loaded steepening pattern remains prominent, mirroring the zero curves.

USD par yields replicate the lower absolute levels and tighter credit structure of USD zero rates (e.g., 5-year differentials of 445–481 bps vs. PYG), with term slopes differing by less than 5 bps from zero curves. The near-equivalence of par–zero relationships across currencies confirms that exchange rate and risk premia affect both representations consistently.

6.3 Historical Dynamics

Figures 3 trace the evolution of 5-year yields across rating categories from January 2023 through December 2025, providing insight into curve stability and responsiveness to market conditions.

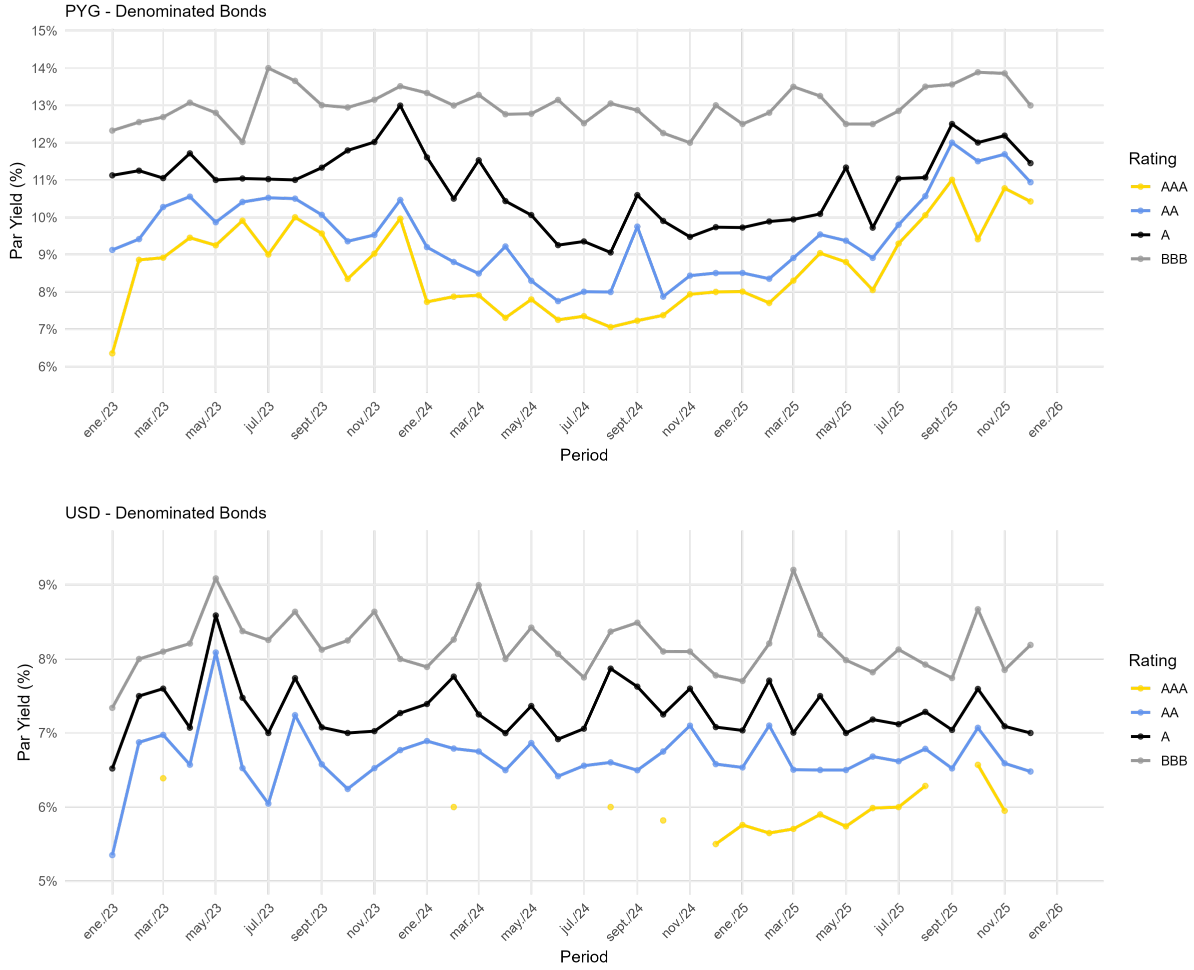

Figure 3: Historical Evolution of 5-Year Par Yields (Jan 2023 – Dec 2025)

6.3.1 PYG Yield Dynamics

The PYG panel (Figure 3) reveals a clear response to monetary policy and credit conditions over the 36-month sample.

The AAA 5-year par yield demonstrates a robust dynamic alignment with the Paraguayan financial rate environment, as illustrated in Figure 10 (see Appendix A.7). The series initiated at 6.35% in January 2023, reached a cyclical floor of 7.06% in August 2024, and peaked at 11.01% in September 2025 (BCP, 2025a). This trajectory exhibits a dual-phase correlation: first, it tracked the BCP's easing cycle from 8.50% to 6.00% through mid-2024 (BCP, 2025a); subsequently, it mirrored the tightening in banking system liquidity, where lending rates climbed from 12.2% in August 2024 to over 16% by late 2025 (BCP, 2025b). This synchronization across policy and commercial benchmarks confirms that the estimated curves reflect genuine macroeconomic drivers rather than fitting artifacts, with the 466 bps total variation underscoring high sensitivity to broader credit cycles.

Credit ordering remains strictly preserved across all periods: BBB > A > AA > AAA, with no curve crossings—a validation of the methodology's hierarchy constraints. Credit spreads exhibit two distinct phases. An early outlier (597 bps BBB–AAA in January 2023) quickly compressed to 369 bps, followed by volatile trading (median 436 bps, std. dev. 101 bps) through mid-2024. From August 2024 onward, spreads tightened markedly, stabilizing at a median of 283 bps in late 2025 (153 bps narrower than the prior phase), suggesting improving BBB fundamentals or market maturation.

All rating curves co-move in response to common factors (correlation 0.462 between AAA and BBB par yields), while credit-specific drivers produce meaningful dispersion—consistent with alternating flight-to-quality and risk-on phases in Paraguay's corporate market.

6.3.2 USD Yield Dynamics

The USD panel displays lower volatility than PYG, with tighter yield ranges and compressed credit spreads reflecting the distinct characteristics of dollar-denominated issuance in Paraguay.

AAA USD coverage is intermittent throughout the sample period, with only 15 of 36 months producing sufficient data for curve estimation. When available, AAA 5-year par yields range narrowly from 5.50% to 6.57% (107 bps variation). The remaining investment-grade ratings show continuous coverage: AA ranges from 5.35% to 8.09% (274 bps), A from 6.52% to 8.59% (207 bps), and BBB from 7.34% to 9.20% (186 bps). These ranges are substantially narrower than PYG AAA variation (466 bps), suggesting that USD yields are partially insulated from local monetary policy cycles and more anchored to global dollar credit conditions.

Credit dispersion remains markedly tighter in USD than in PYG. The BBB–AA median spread of 149 bps compares to PYG BBB–AAA median spread of 446 bps—a difference of nearly 300 basis points. This compression reflects the more homogeneous, higher-quality USD issuer pool, consisting mainly of financial institutions and exporters with natural dollar revenues who access dollar funding markets.

Credit ordering is strictly preserved across all periods with available data, with no instances of curve crossing—validating the methodology's hierarchy constraints in the USD segment as well.

6.4 Comparison with Raw Market Data

Figure 4 overlays the fitted par yield curves with raw anchor points (volume-weighted median yields) in each tenor bucket. Point sizes indicate observation counts on a logarithmic scale, necessary to visualize the wide range of data density (from 2 to 276 transactions per tenor bucket) without obscuring smaller points.

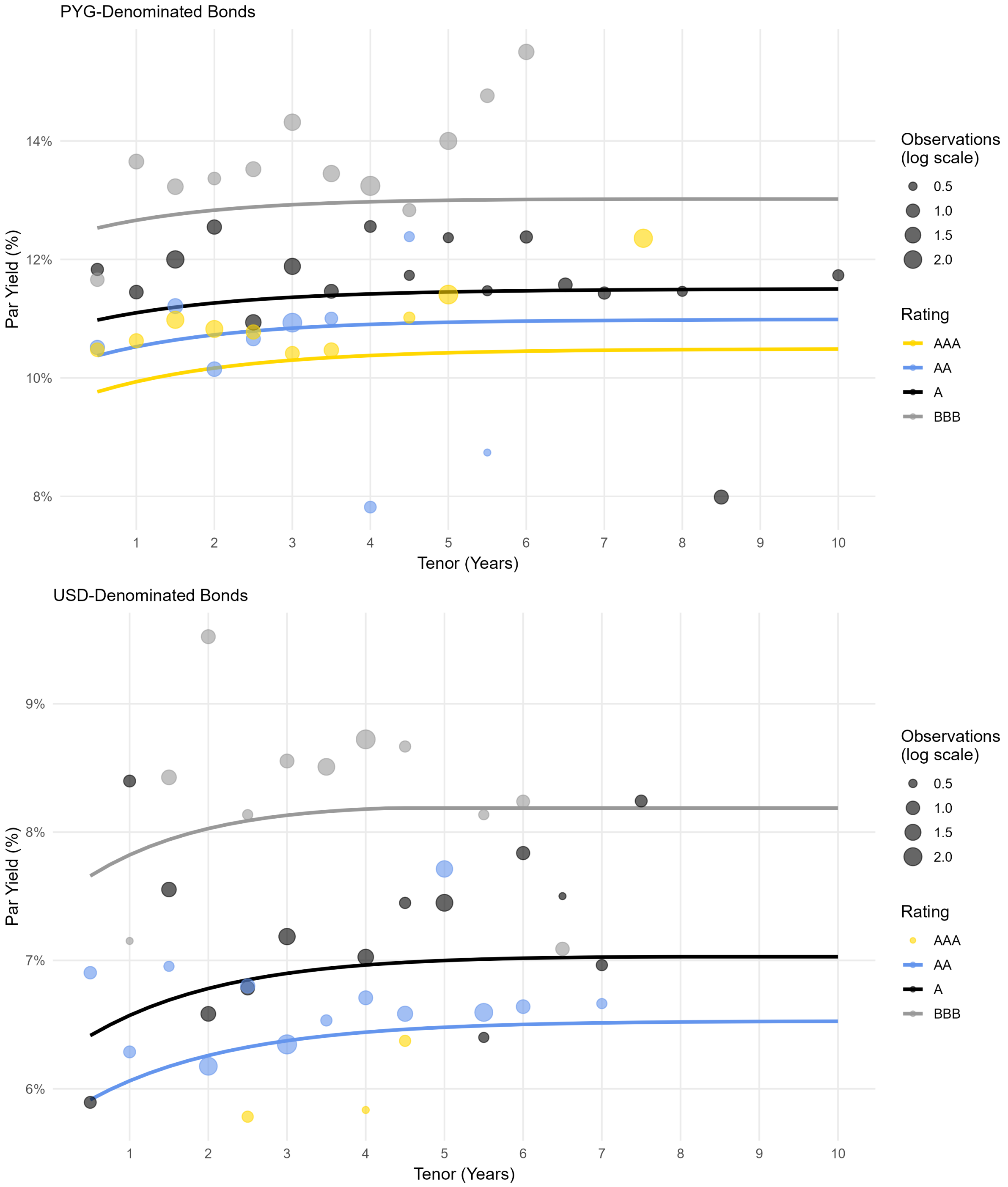

Figure 4: Fitted Yield Curves vs. Raw Anchor Points (December 2025)

The comparison reveals several important features of the methodology's performance:

Close Fit in Data-Rich Segments. In the 0.5–5 year range where anchor points are most abundant, the fitted curves track empirical yields closely. For PYG-denominated bonds, the BBB and A curves pass through or near their respective anchor points across the 0.5–6 year range, with typical deviations of 10–30 basis points. The largest PYG anchor points appear for BBB at the 4-year tenor (276 observations) , AA at the 3-year tenor (214 observations), and AAA at the 5-year tenor (214 observations), confirming that market activity concentrates in these segments.

Smoothing of Sparse Data. In the PYG panel, AAA anchors appear at 10 distinct tenors ranging from 0.5 to 7.5 years, with observation counts per tenor ranging from 4 to 214 (median: 16 observations). The A rating shows the widest tenor coverage with anchors at 17 distinct maturities extending to 10 years, though observation density drops sharply beyond the 5-year tenor (median: 6 observations per tenor). The fitted curves interpolate smoothly across tenor gaps and provide stable estimates even at maturities with limited transaction data.

USD Market Characteristics. The USD panel shows markedly sparser AAA coverage, with anchors appearing at only 3 tenors (2.5, 4, and 4.5 years) and total AAA observations of just 10 transactions across the entire December 2025, month with two hierarchy violations in A anchor at the 0.5 and 5.5-year tenor. This reflects the highly concentrated nature of AAA USD issuance in Paraguay. The AA and A curves show better anchor density (13–14 tenors each) with observation counts ranging from 2 to 204 per tenor. BBB anchors cluster at 11 tenors with a median of 9 observations, maintaining reasonable coverage in the 1–8 year range.

Data Density Variation. Point sizes reveal substantial variation in observation counts across rating-tenor combinations. The PYG total observations by rating are: AAA (571), BBB (565), AA (296), and A (250). For USD: AA (494), BBB (283), A (187), and AAA (10). The dramatic difference in AAA observation counts between currencies—571 for PYG versus just 10 for USD—confirms that top-rated issuers in Paraguay prefer local currency denomination, while dollar issuance concentrates among slightly lower-rated but internationally active credits.

Hierarchy Correction. The raw anchor points reveal why constraint enforcement is necessary. In December 2025 PYG data, several hierarchy violations occur in the raw anchors. Most strikingly, the AA anchor at the 4-year tenor drops to 7.82%—approximately 260 basis points below where AAA yields sit at that maturity. Similarly, the AA anchor at 5.5 years (8.74%) and the A anchor at 8.5 years (7.99%) both fall well below AAA levels. At the short end, the BBB anchor at 0.5 years (11.66%) sits 17 basis points below the A anchor (11.83%). These anomalies likely reflect thin trading in specific tenor-rating cells rather than genuine market pricing. The methodology's hierarchy constraints correct these artifacts, maintaining proper credit ordering (AAA $<$ AA $<$ A $<$ BBB) at all tenors.

Table 3 quantifies the frequency and magnitude of hierarchy corrections across both currencies. For PYG curves, where AAA serves as the base, 78% of AA anchors and 59% of A anchors required upward adjustment, with median corrections of 88 and 45 basis points respectively. Notably, none of the 12 BBB anchors required correction—market prices in this actively traded segment naturally respect credit hierarchy.

For USD curves, the absence of sufficient AAA observations necessitates using AA as the base rating. Corrections are less frequent than in PYG: 50% of A anchors required adjustment (median 26 bps) and only 18% of BBB anchors (median 53 bps). This pattern reflects the more homogeneous, higher-quality USD issuer pool, where market-driven price discovery produces fewer hierarchy violations.

The asymmetry across currencies and ratings confirms that constraints selectively address sparse-data artifacts—concentrated in higher-rated PYG categories with few issuers and institutional-dominated primary placements—while preserving market-driven signals where liquidity is sufficient.

Table 3: Hierarchy Constraint Activation (December 2025)

Currency	Base	Rating	Min Spread	Anchors	Corrected	%	Median (bps)
PYG	AAA	AA	+50 bps	9	7	77.8	88
PYG	AAA	A	+100 bps	17	10	58.8	45
PYG	AAA	BBB	+150 bps	12	0	0.0	–
USD	AA	A	+50 bps	14	7	50.0	26
USD	AA	BBB	+100 bps	11	2	18.2	53

Note: Base rating is the highest-rated curve available for each currency. Min Spread indicates the minimum required spread over the base curve. Median correction shown only for ratings with at least one violation.

6.5 Par vs Zero-Coupon Yield Relationship

Figure 5 displays both par and zero-coupon yields for December 2025, illustrating their systematic relationship across maturities and credit ratings.

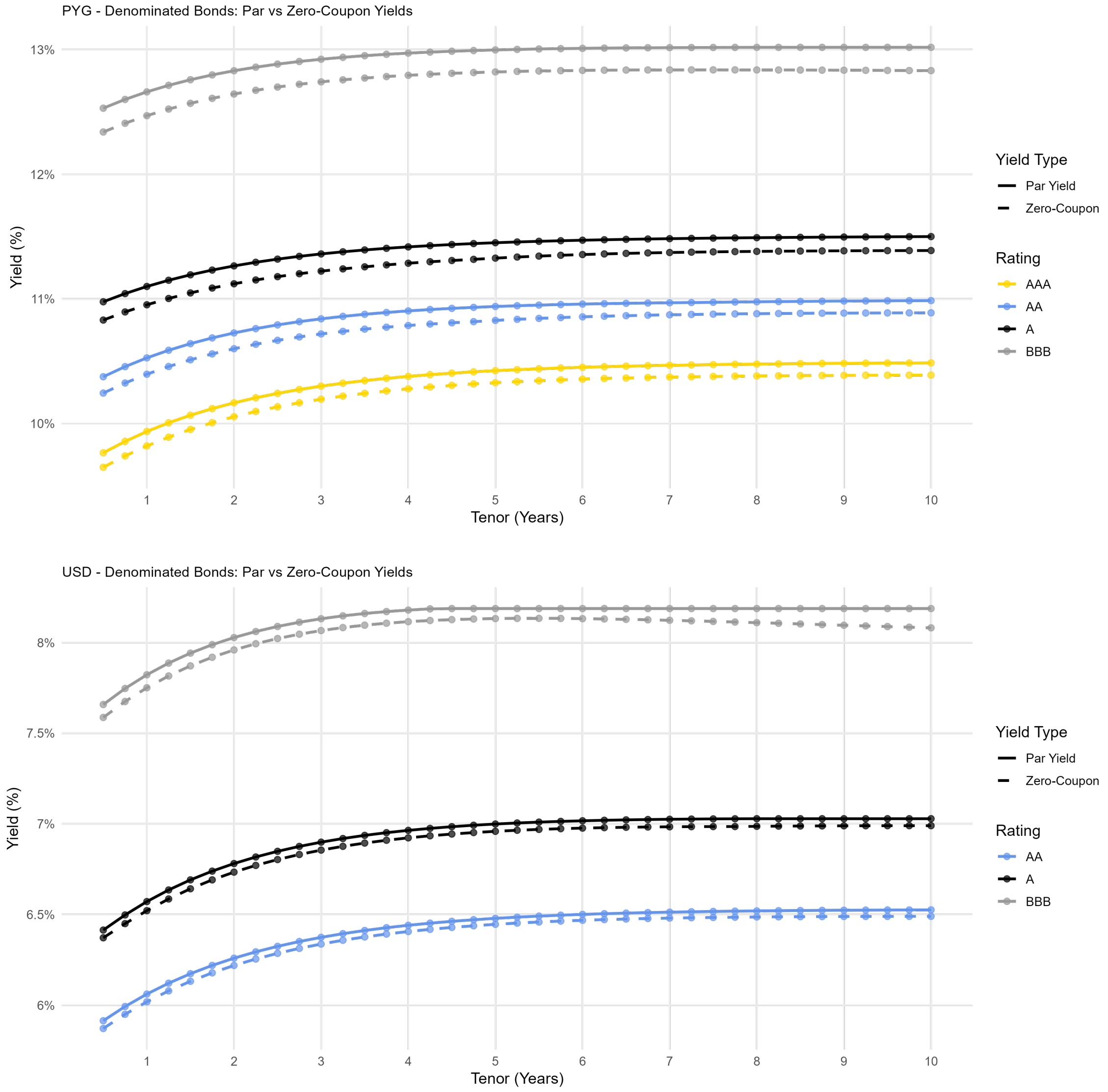

Figure 5: Par vs Zero-Coupon Yield Comparison (December 2025).

Par yields systematically exceed zero-coupon yields by approximately 10–19 basis points for PYG and 3–11 basis points for USD. This differential is not an economic anomaly but reflects differing compounding conventions: zero-coupon yields are continuously compounded (standard for NSS models and derivatives pricing), whereas par yields represent quarterly-compounded coupon rates consistent with local market conventions.

When zero-coupon yields are converted to an equivalent quarterly basis using $r_q = 4(e^{r_{cc}/4} - 1)$[5], the expected relationship for upward-sloping curves—where par yields lie *below* zero rates due to coupon-averaging effects—is restored. The differential increases with credit risk (BBB differentials exceed AAA differentials) because higher absolute yield levels amplify the compounding conversion effect. This pattern confirms the model's internal consistency. A detailed numerical example demonstrating this relationship is provided in Appendix A.5.

6.6 Summary of Key Findings

The yield curve estimation produces economically coherent results across currencies, ratings, and time periods:

- **Currency effects:** PYG yields exceed USD yields by 430–480 bps at comparable ratings and tenors, reflecting expected currency depreciation, inflation differentials, and currency risk premia embedded in local currency bonds.
- **Term structure shapes:** Upward-sloping curves prevail with front-loaded steepening in the 0.5–2 year segment, followed by pronounced flattening beyond 3 years. This pattern is consistent across all rating categories and reflects market expectations of stable long-term conditions.
- **Par-zero relationship:** Par yields systematically exceed zero-coupon yields by 4–19 basis points (PYG) and 3–11 basis points (USD), due to quoting conventions (quarterly vs continuously compounded yields respectively). The differential increases with credit risk due to higher absolute yield levels.
- **Methodology validation:** Comparison with raw anchor points confirms that fitted curves track market yields closely in data-rich segments (typical deviations of 10–30 bps), while correcting four hierarchy violations observed in December 2025 raw data and smoothing noise at sparse tenors.
- **Temporal stability:** Credit ordering persists across the 36-month sample (January 2023–December 2025), with no instances of curve crossing. Spreads demonstrate distinct phases: exceptional January 2023 outlier (597 bps), sustained volatility through July 2024 (median 436 bps), and compression in late 2025 (median 283 bps). The curves respond appropriately to macroeconomic developments, exhibiting 466 basis points of variation (AAA 5-year) aligned with the Central Bank's monetary policy cycle.

7 Published Reference Curves and Usage Guidelines

The Superintendence of Securities publishes monthly reference yield curves derived from this methodology through the Central Bank of Paraguay's institutional website. Both zero-coupon and par yield curves are provided to serve the distinct needs of market participants, supervised institutions, and policymakers.

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[5] where r_{cc} denotes the continuously compounded zero-coupon rate and $r_{quarterly}$ is its quarterly-compounded equivalent

7.1 Curve Types and Consistency

Two curve types are published for each rating-currency combination:

- **Zero-coupon yields:** Continuously compounded spot rates derived directly from the fitted NSS parameters, preserving the exact functional form of the model. Discount factors, forward rates, and other derived quantities maintain theoretical consistency with the fitted parameters.
- **Par yields:** Quarterly-compounded coupon rates subject to isotonic smoothing, guaranteeing monotonicity across all tenors.

The two curve types are not *exactly* consistent: deriving par yields from the published zero-coupon discount factors produces slightly different values than the published par yields. These differences are negligible—typically under 5 basis points—and arise solely from the isotonic smoothing applied to par yields (see Appendix A.5.)

7.2 Zero-Coupon Yield Curves Applications

Zero-coupon yields are published for each credit rating category in both PYG and USD denominations, spanning maturities from 6 months to 10 years at quarterly intervals. These curves serve as reference rates for:

- **Fair value accounting (IFRS 9, IAS 39):** Financial institutions use zero-coupon discount factors to compute present values of contractual cash flows for fair value measurement. IFRS 9 explicitly requires market-observable discount rates; the published zero curves provide defensible, auditable reference rates for this purpose (IASB, 2014).
- **Derivative pricing and hedging:** Interest rate swaps, forward rate agreements, and bond options require zero-coupon discount curves for accurate pricing. Market participants use published curves as input to derivative valuation models, ensuring consistency with observed corporate bond prices.
- **Duration and convexity analysis:** Risk managers compute modified duration, Macaulay duration, and convexity using zero-coupon yields to measure interest rate sensitivity. These metrics are essential for immunization strategies and value-at-risk (VaR) calculations.
- **Pension fund liability valuation:** Insurance companies and pension funds discount future benefit obligations using market-based discount rates. The published zero curves provide appropriate rates for liability measurement under IFRS 17 and local regulatory frameworks.
- **Credit spread analysis:** Analysts decompose corporate yields into sovereign base rates plus credit spreads by comparing corporate zero curves to government yield curves, facilitating credit risk assessment and relative value analysis.

7.3 Par Yield Curves Applications

Par yields yields are published for the same rating-currency-maturity combinations as zero-coupon yields and serve specialized analytical and valuation functions:

- **Primary market pricing:** Issuers and underwriters use par yield curves to determine appropriate coupon rates for new bond offerings. The curves provide market-consensus yields that guide pricing discussions and ensure new issues are competitively priced relative to outstanding securities.

- **Secondary market transparency:** Investors and dealers reference par curves to assess whether individual transaction prices reflect fair value. Material deviations from the reference curve may indicate issuer-specific factors, liquidity premiums, or potential mispricing.
- **Portfolio benchmarking:** Asset managers compare portfolio yields against par curve benchmarks to evaluate relative performance and identify opportunities for yield enhancement or risk reduction.
- **Regulatory reporting:** Supervised entities cite par yields when documenting pricing methodologies for regulatory filings, enhancing consistency and comparability across institutions.

By providing both zero-coupon and par yield curves with transparent methodology, the Superintendence aims to enhance market transparency, support sound valuation practices, and facilitate informed investment decisions in Paraguay's corporate bond market.

8 Limitations and Recommendations

The current framework was developed after evaluating several alternative approaches, including the unextendended Nelson-Siegel specifications with cross-validated decay parameters, fixed lambda values, investment-grade pooling strategies, and various historical data window configurations. Each alternative exhibited limitations—primarily instability with sparse data or inability to maintain credit hierarchy—that motivated the adopted NSS approach with explicit constraints. Details of these alternatives and their shortcomings are documented in Appendix A.6.

8.1 Limitations

The methodology is subject to limitations inherent to Paraguay's emerging bond market:

- **Sparse trading:** Certain rating-currency-tenor combinations have limited observations, particularly for speculative-grade bonds (BB, B) and longer maturities (7+ years). This can result in unstable anchor points or require greater reliance on the parametric model for interpolation. The BB and B curves, while estimated, should be interpreted with particular caution given data limitations.
- **Concentrated issuer base:** As documented in Section 3, a small number of large issuers dominate trading in certain segments. This concentration means that issuer-specific events (rating changes, refinancing activity, idiosyncratic news) can materially affect estimated yields in affected segments.
- **Market segmentation:** The Paraguayan market exhibits segmentation by investor type and distribution channel, which may affect price discovery and yield comparability across transactions. Primary market yields may systematically differ from secondary market yields due to placement premiums or liquidity effects.
- **Observation sparsity:** As documented in Section 3, certain rating-tenor-currency combinations have limited transactions. The methodology interpolates yields at such tenors using NSS functional forms, which may not perfectly reflect idiosyncratic supply-demand dynamics for specific maturities.
- **Credit rating lags:** Published curves rely on credit ratings assigned by authorized rating agencies. Rating changes may lag developments in issuer creditworthiness, particularly for rapidly deteriorating credits. Users should monitor rating agency announcements and adjust valuations accordingly.

- **Market stress periods:** The methodology assumes a "normal" term structure environment through the enforcement of monotonicity, which is based on the dataset's period and ensures an upward-sloping yield curve. However, during periods of aggressive monetary tightening or market turmoil, curves may naturally invert ($y_{\text{short}} > y_{\text{long}}$). Users should therefore interpret results with caution during stress periods, as the model may require a temporary relaxation of the monotonicity constraint to accurately reflect market-driven inversions. Crucially, a distinction must be maintained between the empirical term structure and the structural credit hierarchy. While the slope of the curve (monotonicity) may vary with the economic cycle, the credit hierarchy constraint ($\hat{y}_{AAA} < \hat{y}_{AA} < \cdots < \hat{y}_{B}$) must remain non-negotiable across all tenors. This ensures that even during liquidity shocks or "flight-to-quality" events, the model preserves the fundamental principle that lower-rated debt must carry a risk premium over higher-rated debt of equivalent maturity.

8.2 Recommendations

To enhance the methodology and its applications:

- **Sectoral granularity:** Develop industry-specific yield curves (e.g., agribusiness, manufacturing, and services) once market liquidity and sector issuance increases. While current data scarcity necessitates an aggregate approach to ensure model stability, future refinements should aim to isolate sector-specific risk premia that are currently subsumed within the broader corporate curve.

- **Dynamic spread modeling:** Implement time-varying minimum spread constraints that respond to market conditions and macroeconomic factors. Fixed spreads may be too restrictive during credit market stress or too lenient during benign conditions.

- **Validation framework:** Establish systematic out-of-sample testing using held-out transactions to assess pricing accuracy. Regular backtesting against realized bond prices would quantify methodology performance.

9 Conclusion

This study presents a comprehensive methodology for estimating Paraguay's corporate bond yield curves using the Nelson-Siegel-Svensson model with hierarchical credit constraints. The approach addresses key challenges in emerging market bond pricing through three innovations: event-based yield calculation from actual cash flow schedules, robust anchor construction using volume-weighted medians, and explicit correction of data artifacts to ensure monotonic credit quality ordering across the term structure.

The methodology produces both zero-coupon yields (continuously compounded, as direct NSS output) and par yields (quarterly-compounded, derived from discount factors). The two representations when expressed on the same compounding basis produce the expected theoretical relationship where for upward-sloping curves par yields lie below zero rates, confirming the model's internal consistency.

The estimated December 2025 yield curves reveal a well-ordered credit structure in the Paraguayan corporate bond market. PYG-denominated curves range from 9.76% (AAA 6-month) to 13.02% (BBB 10-year), with credit spreads widening systematically as ratings deteriorate. USD curves display lower absolute yields (5.87–8.19% for AA through BBB) and compressed credit spreads (BBB-AA: 171 bps versus PYG BBB-AAA: 253 bps), reflecting the stronger and more homogeneous credit profile of dollar-denominated issuers—predominantly financial institutions and exporters with natural dollar revenues. Zero-coupon curves exhibit nearly identical patterns at levels 4–19 basis points below their par counterparts.

The curves exhibit economically sensible characteristics validated through multiple lenses. First, both par and zero-coupon term structures slope upward with front-loaded steepening (par yields rise 126 bps from 6 months to 2 years for AAA, then flatten to 44 bps from 2 to 10 years; zero yields show 135 bps and 38 bps respectively). Second, monotonic credit ordering is preserved at all maturities across all 36 historical months in both yield representations, with no instances of curve crossing. Third, fitted curves track raw market data closely in liquid segments (deviations of 10–30 bps) while correcting hierarchy violations that appear in sparse-data segments. Fourth, temporal dynamics align with macroeconomic fundamentals: the 466 basis point variation in AAA par yields (6.35–11.01%) mirrors the Central Bank's policy rate cycle and banking system rate movements.

These properties are not guaranteed by raw market data alone. The December 2025 data reveals four hierarchy violations in raw anchor points—including an AA anchor at the 4-year tenor falling to 7.8%, well below AAA levels. The methodology's hierarchical constraints correct these anomalies while preserving empirical fit, producing curves that satisfy fundamental credit market principles even in data-sparse environments.

The resulting yield curves—both par and zero-coupon—provide economically coherent benchmarks for securities valuation, portfolio management, and regulatory supervision. The framework is implemented in a fully automated monthly processing pipeline that handles transaction data, computes event-based yields, constructs weighted anchor points, estimates NSS parameters with credit constraints, and produces complete term structures for both yield representations across all rating-currency combinations.

While the methodology is tailored to Paraguay's market characteristics—including sparse long-dated observations, concentrated issuer pools in certain segments, and occasional data artifacts requiring correction—the underlying principles of robust estimation, constraint enforcement, and event-based pricing are broadly applicable to emerging markets facing similar challenges. Future extensions may incorporate credit transition dynamics, develop spread forecasting models, and establish formal validation frameworks using holdout samples or cross-sectional pricing errors.

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A Appendix: Technical Specifications and Supplementary Data

A.1 Market Characteristics

A.1.1 Treatment of Zero-Coupon Instruments

Zero-coupon instruments, including Bonos Bursátiles de Corto Plazo (BBCP), are included in the yield curve estimation despite their small market share. As described in Table A-1, zero-coupon instruments represent 0.67% of transactions (414 observations) and 0.77% of trading volume over the analysis period, with an average maturity of 0.45 years. See for detailed statistics. Their inclusion is motivated by two considerations:

First, zero-coupon instruments provide valuable yield observations at the short end of the maturity spectrum, where coupon-bearing bonds may be sparse. The average maturity of 0.45 years positions these instruments precisely in the sub-1-year segment that is often underrepresented in corporate bond trading.

Second, the event-based yield calculation methodology handles zero-coupon bonds seamlessly. A zero-coupon bond is simply a special case where the only cash flow is principal repayment at maturity. The yield solver correctly computes the discount rate that equates the purchase price to the present value of this single payment. The 100% success rate in yield computation for these 414 transactions confirms their compatibility with the methodology.

The yields from zero-coupon instruments are included in anchor point construction on equal footing with coupon-bearing bonds, subject to the same winsorization and weighting procedures. Their short maturities mean they primarily influence the leftmost portion of the estimated curves.

Table A-1: Zero-Coupon Instrument Statistics
Jan 2023 – Dec 2025

Statistic	**Value**
Number of Transactions	414
Share of Total Transactions	0.67%
Trading Volume (Gs)	318.3 billion
Share of Total Volume	0.77%
Average Maturity (years)	0.45
Yield Computation Success Rate	100%

A.1.2 Issuer Concentration

Figure 6 illustrates the concentration of trading volume among issuers in Paraguay's corporate bond market.

The top issuers consist primarily of financial institutions (Sudameris Bank, Banco Itaú Paraguay, Banco GNB Paraguay, Banco Familiar, Banco Continental, Zeta Banco, Ueno Bank) and select corporate names from telecommunications (Telecel SAE) and agribusiness (Frigorífico Concepción, Grupo Vazquez).

This composition reveals two important patterns. First, financial sector issuers predominate among the most active names, consistent with banks' ongoing funding needs and their role as frequent capital market participants. Second, the corporate issuer base, while present, remains relatively thin, with only a handful of non-financial corporations achieving significant trading volumes. The remaining 92 issuers share 39.6% of volume, implying an average share of just 0.4% per issuer—highlighting the long tail of smaller, less liquid names.

For yield curve estimation, this concentration pattern implies that certain rating-tenor segments may be dominated by a small number of issuers, potentially introducing issuer-specific

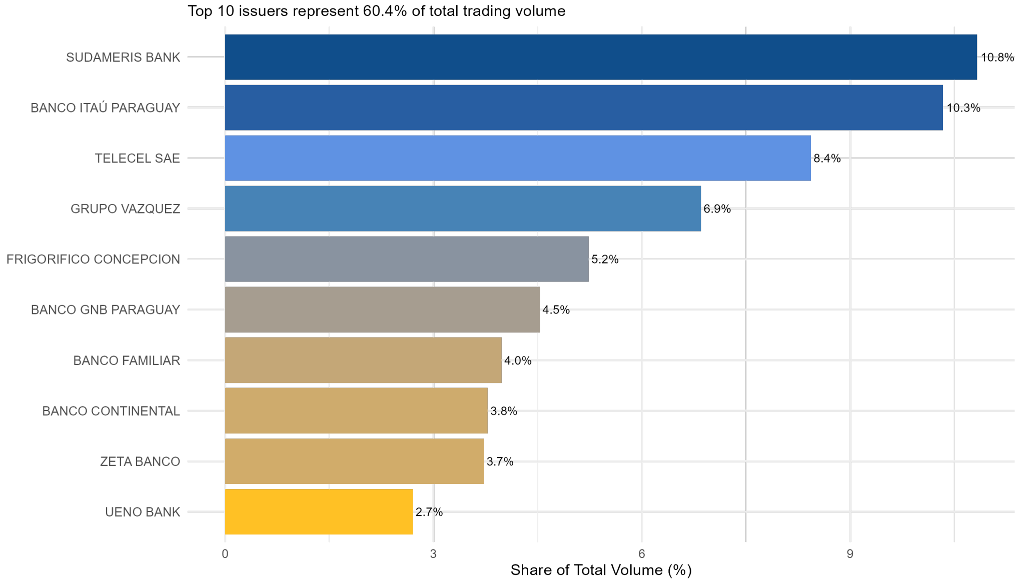

Figure 6: Issuer Concentration: Top 10 Issuers by Trading Volume (Jan 2023–Dec 2025)

effects into aggregate yield measures. The robust estimation techniques employed in Section 5 are designed to mitigate such influences.

A.1.3 Primary vs. Secondary Market Activity

Figure 7 presents the distribution of trading activity between primary and secondary markets.

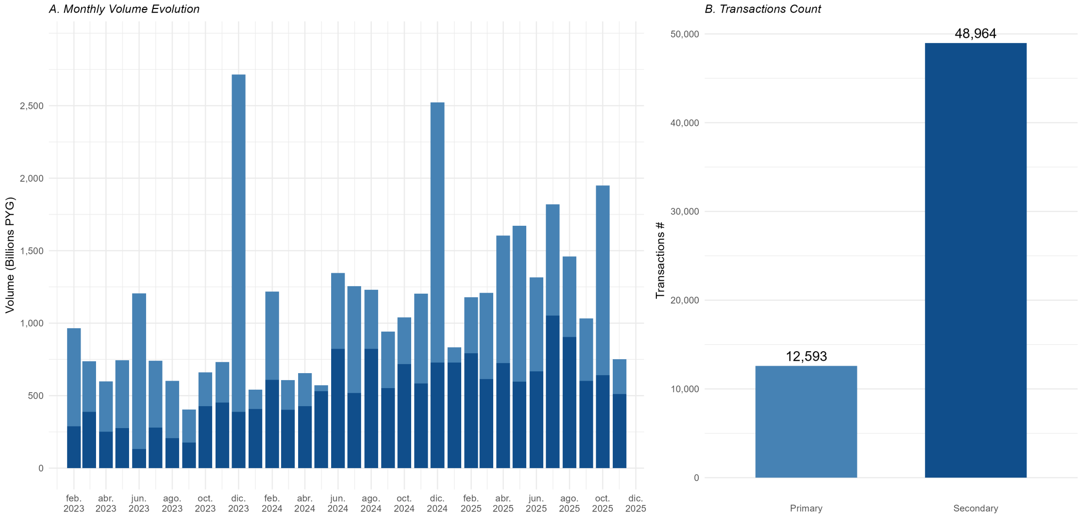

Figure 7: Primary vs Secondary Market Activity (Jan 2023 – Dec 2025)

The secondary market accounts for 79.5% of all transactions but only 46.6% of total volume. Conversely, the primary market represents just 20.5% of transactions yet captures 53.4% of volume. This stark asymmetry reflects fundamental differences in how the two markets operate.

Primary market transactions—new issuance—tend to be large institutional placements where substantial volumes change hands in single transactions. While the secondary market is more active in terms of frequency (recording 3.9 times as many transactions as the primary market), the primary market is characterized by much larger individual deal sizes. On average, a single primary market transaction moves approximately 4 times more capital than a secondary market trade. The latter reflects that investors in new issuances, typically institutional and qualified investors, commit significant capital in primary offerings and then largely hold positions to maturity.

Secondary market activity, by contrast, consists of more numerous but smaller transactions. The prevalence of transaction count (79.5%) over volume (46.6%) indicates active trading in relatively modest sizes, often reflecting portfolio rebalancing, liquidity needs, or tactical positioning rather than large-scale investment decisions.

This buy-and-hold behavior has important implications for yield curve estimation. Secondary market prices, while more frequent, may reflect liquidity-driven rather than fundamental valuation considerations. The methodology accounts for this by volume-weighting observations, ensuring that larger, presumably more informative transactions receive appropriate influence in anchor point construction.

A.1.4 Trading Activity Across Maturities

Figure 8 documents the distribution of trading activity across the maturity spectrum—perhaps the most consequential market characteristic for yield curve estimation.

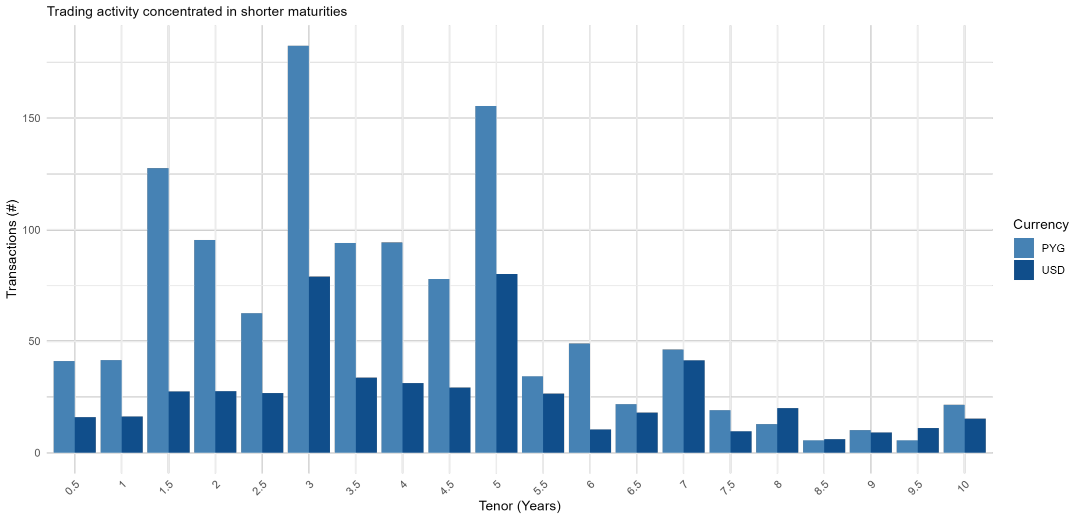

Figure 8: Average Monthly Transactions by Tenor Group (Jan 2023 – Dec 2025)

Trading activity is heavily concentrated in the 1.5–5 year maturity segment. The 3-year tenor shows the highest activity for PYG (approximately 175 average monthly transactions), followed by the 5-year tenor (approximately 155 monthly transactions) and the 1.5-year tenor (approximately 125 monthly transactions). USD activity peaks at the 3-year and 5-year tenors with approximately 80 monthly transactions each. The short end (0.5–1 year) shows moderate activity with approximately 40–45 transactions monthly in PYG and 15–20 in USD.

Longer maturities exhibit progressively sparser trading. The 6–7 year segment averages 35–50 transactions monthly in PYG and 20–40 in USD, while the 8+ year segment drops to approximately 5–25 transactions monthly. At the long end, monthly averages fall below 25 transactions per currency, implying fewer than 6 transactions per week spread across multiple rating categories—a level of sparsity that challenges conventional curve-fitting approaches.

This maturity distribution pattern motivates several methodological choices: the use of robust estimation techniques that perform well with limited data, the weighting schemes that up-weight scarcer long-tenor observations, and the minimum observation thresholds that balance reliability against practical data availability.

A.1.5 Transaction Density by Rating and Tenor

Figure 9 presents a heatmap of average monthly transactions across the rating-tenor grid, revealing the granular data availability that underlies yield curve estimation.

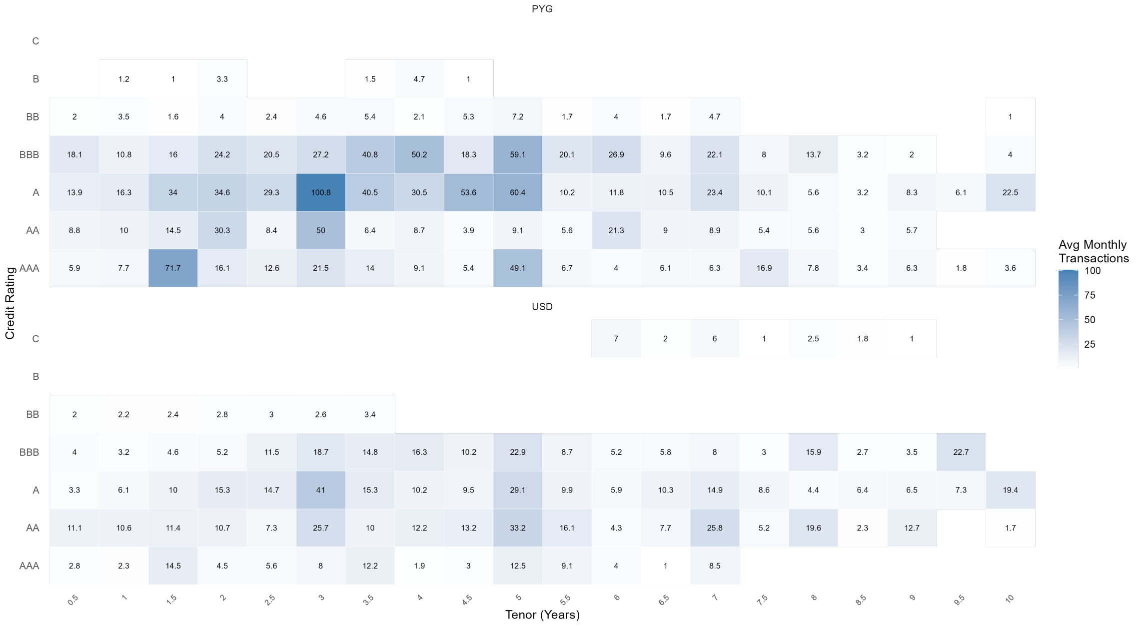

Figure 9: Average Monthly Transactions by Rating and Tenor (Jan 2023 – Dec 2025)

The highest transaction densities occur in the A rating category at medium tenors, with the 3-year A-rated PYG cell reaching approximately 101 average monthly transactions—the highest density in the entire grid. The BBB category also shows strong activity, particularly at the 5-year tenor (59 monthly transactions) and 4-year tenor (50 monthly transactions). Investment-grade categories (AAA, AA, A, BBB) generally show adequate coverage across most tenor buckets, though with notable variation.

For PYG, the AAA category shows concentrated activity at specific tenors: the 1.5-year bucket averages 72 monthly transactions while the 5-year bucket averages 49 monthly transactions, but intermediate tenors show much lighter coverage (9–21 monthly transactions). The AA category displays moderate activity across tenors, with peaks at the 2-year (30 monthly transactions) and 3-year (50 monthly transactions) points.

Speculative-grade ratings (BB, B) exhibit substantially sparser coverage. The BB category averages fewer than 8 transactions per month across most tenor buckets, with peak activity at the 5-year tenor (7 monthly transactions). B-rated bonds show even lower activity, with most cells averaging 1–5 transactions monthly and several tenor buckets showing no activity at all.

For USD, the pattern differs somewhat. The A-rated 3-year bucket shows the highest activity (41 monthly transactions), followed by AA at the 5-year tenor (33 monthly transactions) and BBB at the 5-year tenor (23 monthly transactions). The AAA USD segment has limited coverage, with most tenors averaging fewer than 15 monthly transactions, explaining the intermittent availability of the USD AAA curve in certain estimation periods.

The lowest tier, the C rating, shows near-total illiquidity in both currencies, with only a few isolated transactions in the USD 6–9 year range, rendering it unsuitable for term structure estimation. Overall, the heatmap reveals maturity-specific patterns within rating categories. Such “gaps” in the tenor grid underscore the importance of the NSS model’s ability to interpolate smoothly across maturities where direct observations are sparse.

A.1.6 Coupon Payment Frequency

Table A-2 documents the complete distribution of coupon payment frequencies in the dataset.

Table A-2: Distribution of Coupon Payment Frequencies
Jan 2023 – Dec 2025

Payment Frequency	**% Transactions**	**% Bonds**	**% Volume**
Trimestral (Quarterly)	69.7	63.6	46.7
Mensual (Monthly)	19.1	13.9	15.2
Semestral (Semi-annual)	8.0	13.0	32.5
Variable	1.3	5.2	0.5
Cupón Cero (Zero Coupon)	0.7	1.5	0.8
Bimestral (Bi-monthly)	0.6	0.5	0.2
Anual (Annual)	0.5	1.5	3.9
Bianual (Bi-annual)	0.2	0.7	0.1
Al Vencimiento (At Maturity)	0.0	0.3	0.2
Total	**100.0**	**100.0**	**100.0**

Quarterly coupon payments dominate the market, accounting for 69.7% of all transactions, 63.6% of unique bonds, and 46.7% of trading volume. The next most common frequency is monthly payments (19.1% of transactions), followed by semi-annual (8.0%). Therefore, while monthly and semi-annual bonds contribute meaningfully to trading activity, quarterly payments represent the clear modal convention. The par yield curves produced by this methodology serve as benchmarks; individual bond valuations can adjust for different payment frequencies as needed.

A.1.7 Instrument Types by Sector

Table A-3 presents median yields for the 2–5 year tenor segment by rating, sector, and instrument type for PYG-denominated bonds.

Table A-3: Median Yields by Sector and Instrument Type: PYG, Medium Tenor (2–5Y)
Jan 2023 – Dec 2025

Rating	Sector	Bono	Bono Fin.	Bono Sub.	Avg. Mo. Tr.
AAA	Sector Financiero	6.40%	7.71%	8.16%	14.0
	Servicios	10.56%	—	—	59.8
AA	Agricultura	9.24%	—	—	2.6
	Comercio al por Mayor	10.76%	—	—	3.6
	Sector Financiero	—	9.11%	8.52%	49.2
	Servicios	8.32%	—	—	1.5
A	Actividades Inmobiliarias	12.00%	—	—	3.3
	Agricultura	11.59%	—	—	41.4
	Comercio al por Mayor	10.97%	—	—	53.9
	Ganadería	12.43%	—	—	104.0
	Industria	9.87%	—	—	18.7
	Sector Financiero	10.54%	10.21%	8.67%	71.2
	Servicios	12.02%	—	—	17.4
BBB	Actividades Inmobiliarias	14.20%	—	—	5.9
	Agricultura	13.66%	—	—	105.0
	Comercio al por Mayor	13.37%	—	—	74.3
	Construcción	13.23%	—	—	8.1
	Ganadería	14.22%	—	—	6.3
	Sector Financiero	—	—	9.72%	1.2
	Servicios	13.65%	—	—	12.1

Note: The "Avg. Mo. Tr." column represents the average number of monthly transactions recorded over the 36-month sample period. Within the financial sector, multiple instrument types coexist with modest yield differences, whereas cross-sector variation within the same rating category remains the primary driver of yield spreads. All yields are expressed as Effective Annual Yields (EAY).

Within the financial sector—the only sector where multiple instrument types coexist—median yield differences range from 20 to 180 basis points depending on rating and tenor, with most differences below 150 basis points. The slightly higher yield observed for Bono Subordinado compared to Bono Financiero in some instances (e.g., a spread of 146 bps in the AAA segment) is consistent with the subordination of claims in liquidation.

In contrast, cross-sector yield variation within the same rating category is considerably larger. For A-rated PYG bonds, median yields range from 9.87% (Industria) to 12.43% (Ganadería)—a spread of 256 basis points. For BBB-rated bonds, the cross-sector range reaches 450 basis points. The dominance of primary sectors is reflected in transaction counts, where Ganadería and Agricultura account for the highest liquidity density (averaging 104.0 and 105.0 monthly transactions for A and BBB ratings, respectively).

A.2 Notation and Definitions

This section establishes the mathematical notation used throughout the paper. We distinguish carefully between transaction-level observed yields, model-implied spot rates, and derived par yields.

Table A-4: Summary of Mathematical Notation

Symbol	**Definition**
Model Outputs	
$r(\tau; \boldsymbol{\theta})$	Continuously compounded spot (zero-coupon) rate at maturity τ
$D(\tau)$	Discount factor: $D(\tau) = \exp\left(-r(\tau; \boldsymbol{\theta}) \cdot \tau\right)$
$c(\tau)$	Par yield at maturity τ (quarterly compounded)
$\hat{r}_k(\tau)$	Fitted spot rate curve for rating category k
$\hat{c}_k(\tau)$	Fitted par yield curve for rating category k
Transaction-Level Yields	
$\tilde{y}$	Effective annual yield (EAY) solved from transaction price
$y_i^{(f)}$	Yield-to-maturity for transaction i at compounding frequency f
y_i	Transaction-level YTM (shorthand for $y_i^{(f)}$)
y_i^w	Winsorized YTM for transaction i
Anchor Points	
$\bar{y}_\tau$	Anchor point: volume-weighted median of $\{y_i^w\}$ at tenor τ
B_τ	Set of transactions in tenor bucket τ
Model Parameters	
$\boldsymbol{\theta}$	NSS parameter vector $(\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2)^\top$
$\hat{\boldsymbol{\theta}}$	Estimated parameter vector
β_0	Level parameter (long-run yield)
β_1	Slope parameter (short-term spread)
β_2, β_3	Curvature parameters
λ_1, λ_2	Decay parameters
s_k	Minimum spread for rating k over AAA
Optimization	
$\mathcal{L}(\boldsymbol{\theta})$	Objective function
$L_H(\cdot)$	Huber loss function
e	Residual: $e = \bar{y}_\tau - \hat{c}(\tau)$
w_i	Transaction weight: $w_i = w_i^{vol} \cdot w_i^{mat}$
$\hat{c}^{iso}(\tau)$	Isotonically smoothed par yield
Time and Structure	
τ	Time to maturity (years)
t_s	Settlement date
t_i	Cash flow payment date i
f	Coupon frequency (payments per year)
Δ_i	Day count fraction for period i
CF_t	Cash flow at time t
P_{dirty}	Dirty price per 100 units of face value

The notation follows standard conventions in term structure literature: r denotes spot rates (Diebold & Li, 2006), c denotes par yields, and Greek letters (ρ, $\boldsymbol{\theta}$) denote quantities estimated from or applied to individual observations. Hats ($\hat{\cdot}$) indicate fitted values, bars ($\bar{\cdot}$) indicate central tendency measures, and tildes would indicate preliminary or unadjusted estimates where applicable.

A.3 Yield Solver: Mathematical Specification

This part of the appendix provides the complete mathematical framework for the event-based yield calculation methodology.

A.3.1 Outstanding Principal

For amortizing bonds, the outstanding principal at settlement date t_s is less than the original face value due to prior principal repayments. Let $\{A_t\}$ denote the sequence of amortization payments recorded in the corporate events schedule, expressed as percentages of the original face value. The outstanding principal per 100 units of original face value is:

$$P_{out}(t_s) = 100 - \sum_{t \leq t_s} A_t \tag{16}$$

where the summation includes all amortization payments made on or before the settlement date. For bullet bonds (no intermediate amortization), $P_{out}(t_s) = 100$ throughout the bond's life.

A.3.2 Accrued Interest

Accrued interest represents the portion of the next coupon payment that has accumulated from the previous payment date to the settlement date. Let t_{prev} denote the most recent coupon payment date on or before settlement, and t_{next} the next scheduled coupon payment date. If I_{next} denotes the coupon payment amount at t_{next} (per 100 units of original face value), accrued interest is computed via linear day-count interpolation:

$$AI(t_s) = I_{next} \cdot \frac{t_s - t_{prev}}{t_{next} - t_{prev}} \tag{17}$$

This linear accrual convention is consistent with the Actual/365 day count basis used in the Paraguayan market.

A.3.3 Price Conversion

Market prices are quoted as clean prices per 100 units of *outstanding* principal. For yield calculation, we require the dirty price per 100 units of *original* face value:

$$P_{dirty} = P_{clean} \cdot \frac{P_{out}(t_s)}{100} + AI(t_s) \tag{18}$$

where P_{clean} is the quoted clean price per 100 outstanding.

A.3.4 Brent's Method

We solve for y using Brent's Method, a robust root-finding algorithm that combines bisection, secant method, and inverse quadratic interpolation. To ensure convergence across a wide range of market conditions (including distressed debt or high-yield environments), the solver employs a dynamic bounding strategy:It initializes a search interval $[l, u]$.If $f(l)$ and $f(u)$ have the same sign (indicating no root is trapped), the algorithm iteratively expands the bounds ($l = l - 0.5$, $u = u + 0.5$) until a sign change is detected or the search limit is reached.Once a root is bracketed, the algorithm converges to the solution with a tolerance of 1×10^{-5}.

A.4 NSS Optimization: Technical Specifications

This part of the appendix provides complete mathematical specifications for the NSS parameter optimization described in Section 5.3.

A.4.1 Huber Loss Function

Huber loss is defined as:

$$L_H(e) = \begin{cases} \frac{1}{2}e^2 & \text{if } |e| \leq k \\ k \cdot \left(|e| - \frac{1}{2}k\right) & \text{if } |e| > k \end{cases} \tag{19}$$

where $e = \bar{y}_\tau - \hat{c}(\tau; \boldsymbol{\theta})$ is the residual between the anchor yield and the model-implied par yield. The transition point k is set at 1.345 times the median absolute deviation of the anchor yields, a standard robust statistics threshold that achieves approximately 95% efficiency relative to OLS when data is truly normal, while providing stronger protection against outliers.

A.4.2 Monotonicity Penalty

The monotonicity penalty is defined as:

$$P_{mono}(\boldsymbol{\theta}) = \sum_{j=1}^{M-1} \left[\max\left(0, -\Delta\hat{c}_j\right)\right]^2 \tag{20}$$

where $\Delta\hat{c}_j = \hat{c}(\tau_{j+1}) - \hat{c}(\tau_j)$ is the yield change between adjacent grid points, M denotes the number of grid points on the tenor grid, and $j \in \{1, \ldots, M-1\}$ indexes adjacent pairs. The penalty is evaluated on a dense tenor grid and penalizes any decreases in yield as maturity increases.

This approach draws on the literature on shape-constrained curve fitting. Hagan & West (2006) discuss monotonicity as a desirable property for yield curve interpolation, noting that non-monotonic curves can produce economically implausible forward rates. De Pooter (2007) examines regularization techniques for Nelson-Siegel class models.

A.4.3 Level Anchor

The level anchor penalty centers the curve near observed market yields at the 5-year tenor:

$$P_{level}(\boldsymbol{\theta}) = \left(\hat{c}(5; \boldsymbol{\theta}) - \bar{y}_{5Y}\right)^2 \tag{21}$$

where $\bar{y}_{5Y}$ is the median observed anchor yield near the 5-year tenor. The 5-year maturity is selected based on Diebold & Li (2006), who demonstrated that medium-term tenors best capture the "level" factor of the yield curve.

A.4.4 Band Constraint

The band constraint prevents extrapolation beyond observed yields:

$$P_{band}(\boldsymbol{\theta}) = \sum_{\tau} \left[\max(0, \hat{c}(\tau) - q_{90} - 0.01)^2 + \max(0, q_{10} - 0.01 - \hat{c}(\tau))^2\right] \tag{22}$$

where q_{10} and q_{90} are the 10th and 90th percentiles of observed anchor yields. The 1% buffer allows modest extrapolation while preventing extreme curve shapes.

A.4.5 Penalty Weights

The penalty weights in the objective function (Equation 14) are calibrated as:

Table A-5: Penalty Weight Calibration

Symbol	**Penalty Term**	**Weight**
α_1	Monotonicity (P_{mono})	1,000
α_2	Level anchor (P_{level})	5,000
α_3	Band constraint (P_{band})	1,000

Weights were selected via sensitivity analysis to balance fitting accuracy against constraint enforcement. Results are robust to variations within one order of magnitude—multiplying or dividing any weight by a factor up to 10 produces qualitatively similar results.

A.4.6 Parameter Bounds

Parameters are estimated using the L-BFGS-B algorithm with box constraints:

Table A-6: Parameter Bounds for NSS Optimization

Parameter	**Lower Bound**	**Upper Bound**
β_0	$0.9 \cdot y_{min}$	$1.1 \cdot y_{max}$
β_1	-0.10	0.10
β_2	-0.10	0.10
β_3	-0.05	0.05
λ_1	0.5	8.0
λ_2	1.0	15.0

The bounds for β_0 are data-driven, set to 0.9–1.1 times the observed yield range in each segment. The bounds for β_1, β_2, and β_3 ensure that coefficient values outside these ranges—which would produce extreme curve shapes rarely observed in practice—are excluded. The λ bounds ensure decay factors produce economically meaningful curve dynamics, with curvature effects concentrated at plausible maturities.

A.5 Par vs Zero-Coupon Yield Relationship: Compounding Reconciliation

Par yields exceed zero-coupon yields by 3–18 basis points (Table A-7). This apparent anomaly reflects differing compounding conventions rather than an economic inconsistency.

Zero-coupon yields are continuously compounded ($D(t) = e^{-r_{cc} \cdot t}$), while par yields are quarterly-compounded coupon rates. To compare them on the same basis, convert the continuous rate to its quarterly-equivalent:

$$r_{quarterly} = 4 \times \left(e^{r_{cc}/4} - 1\right) \tag{23}$$

For example, the PYG AAA 5-year zero rate of 10.33% (continuous) converts to 10.46% (quarterly)—a 13 basis point increase. Since the par yield (10.42%) lies between these values, the apparent anomaly is explained: on the same compounding basis, $y_{par} < y_{zero}$ as theory predicts.

Table A-7: Par vs Zero-Coupon Yields: Compounding Reconciliation (5-Year Tenor, December 2025)

Ccy	**Rating**	**Zero (cc)**	**Zero (q)**	**Par**	**Par − Zero(cc)**	**Par − Zero(q)**
PYG	AAA	10.33%	10.46%	10.42%	+9.7	−3.8
PYG	AA	10.82%	10.97%	10.94%	+11.2	−3.6
PYG	A	11.32%	11.48%	11.45%	+12.4	−3.8
PYG	BBB	12.77%	12.98%	12.95%	+17.7	−3.1
USD	AA	6.45%	6.50%	6.48%	+3.3	−1.9
USD	A	6.96%	7.02%	7.00%	+4.0	−2.1
USD	BBB	8.13%	8.22%	8.19%	+5.5	−2.8

Note: All values in the final column are negative, confirming $y_{par} < y_{zero}$ when both use quarterly compounding. Higher yield levels produce larger compounding conversion effects, explaining why PYG and lower-rated bonds show larger differentials.

A.6 Methodological Robustness Checks

A.6.1 Nelson-Siegel with Cross-Validated Lambda

An initial approach employed the simpler Nelson-Siegel model (three parameters) with the decay parameter λ selected via grid search and k-fold cross-validation. For each candidate λ within a specified range (0.05 to 0.5), the model was fit using robust linear regression (RLM), and out-of-sample RMSE was computed. While this approach provided reasonable fits for certain segments, it exhibited notable instability with sparse datasets—sometimes producing extreme or implausible curve shapes. Moreover, the three-parameter specification could not adequately capture the double-humped patterns occasionally observed in corporate credit curves.

A.6.2 Fixed Lambda Value

A second approach tested the use of a fixed λ value chosen through internal calibration across all segments. While enhancing stability, this introduced subjectivity and could not account for dynamic differences between rating and currency groups.

A.6.3 Investment Grade Pooling

To address data sparsity, an approach was tested that grouped all investment-grade ratings (AAA, AA, A, BBB) and treated them as a single yield curve. However, this still produced distortions showing reverse yield curves at certain maturities, as the underlying data heterogeneity across rating categories introduced noise that the single-curve estimation could not properly resolve.

A.6.4 Historical Data Window Variations

Various data window configurations were tested. Using the entire 12-month historical data with time-weighted averaging (as opposed to regression over period-by-period weighted average points) still produced reverse curve tendencies at longer maturities. Similarly, restricting to the most recent 3 months to capture more current market conditions did not resolve the convergence issues at long maturities.

These experiences underscored the necessity of the current approach: joint optimization of all NSS parameters with explicit hierarchy constraints, robust loss functions, and multiple penalty terms. The adopted framework directly addresses the shortcomings identified in previous methods while remaining grounded in established econometric practice.

A.7 Macroeconomic Validation Analysis

This section provides a detailed visual and technical validation of the estimated corporate yields against official monetary policy benchmarks.

Figure 10 presents the longitudinal comparison between policy, banking, and estimated corporate rates.

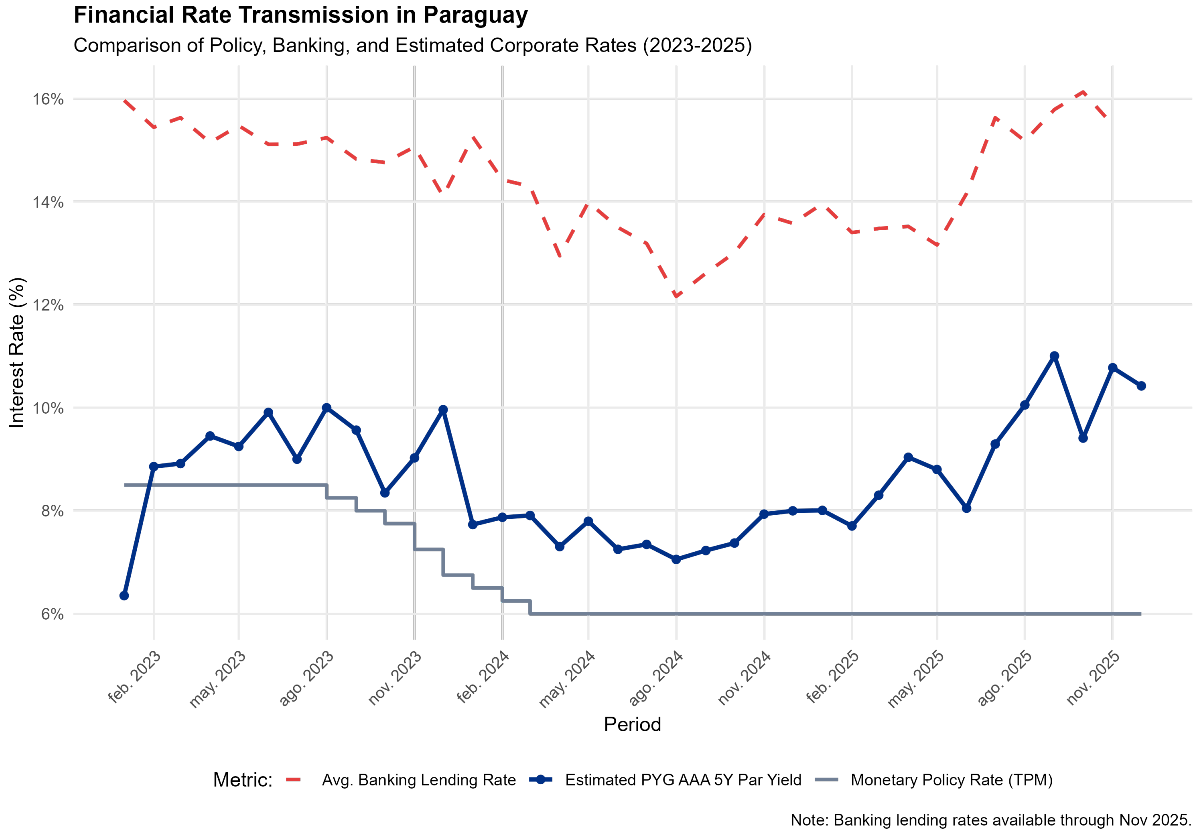

Figure 10: Financial Rate Transmission: TPM, Banking Lending Rates, and Estimated AAA 5Y Par Yield (2023–2025).

Extended Analysis of Rate Dynamics: The enriched dataset reveals a complex transmission mechanism during the 36-month period:

- **The Policy Floor (2023–2024):** During the easing cycle, corporate yields followed the TPM downward. In August 2024, when the TPM reached 6.00% and lending rates hit a period low of 12.2%, the AAA yield reached its minimum of 7.06%, maintaining a consistent spread above the risk-free proxy (BCP, 2025a,b).
- **Commercial Decoupling (2025):** While the TPM remained anchored at 6.00%, both banking lending rates and corporate yields pivoted upward. The AAA yield's rise to 11.01% in September 2025 mirrors the surge in banking rates to 16.1% in October 2025 (BCP, 2025b).
- **Market Sensitivity:** This triangulation validates that the NSS framework captures systemic shifts in credit costs. The corporate curve functions as an efficient "middle-tier" rate, sensitive to both Central Bank policy signals and commercial bank liquidity pressures.

The preservation of the credit hierarchy throughout these volatile shifts ensures that the model remains a reliable benchmark for valuation, even when market rates decouple from the overnight policy target.