Models of Balance of Payments Crises with Capital Controls

Emilio Espino, Afonso Gauna and Andy Neumeyer
Universidad Torcuato Di Tella

UDESA

September 12th, 2023

Inconsistent monetary and exchange rate policies

Some countries want to finance deficits by printing money and, to keep inflation low, adopt an unsustainable fixed exchange rate/crawling peg.

domestic credit growth $\theta > \epsilon$ rate of devaluation

Free Capital Mobility: speculative attack on the BoP (Krugman, 1979)

Techniques to delay the collapse of an unsustainable fixed exchange rate regime

Models of balance of payments crisis with capital controls

- with free trade (Bretton Woods?)
- with import restrictions (emerging economies)

A Model of BoP Crisis with Capital Controls with Free Trade

- Capital controls may delay the collapse of the fixed exchange rate v Krugman's case.
- Monetary approach to the BoP (Frenkel and Johnson, 1976).
  - Δ money demand (interest rate) - deficit monetization $= \Delta$ Reserves $=$ CA balance
- While the fixed exchange rate regime survives;
  - inflation is low,
  - consumption is above steady state and increasing (collapses after devaluation),
  - the shadow exchange rate anticipates devaluation (Sargent and Wallace (1973),Lucas (1978))
  - real interest rates rise above international rates along with consumption boom (Frenkel and Razin, 1989)
  - Delayed Monetization (Sargent and Wallace, 1981) extends the low inflation regime at the cost of more inflation later.
- The regime ends with an **anticipated devaluation** when reserves are zero (Park and Sachs, 1987).

A Model with Capital Controls with Import Restrictions

Introduce import restrictions so that Δ Reserves $= 0$ forever.

- The fixed exchange rate regime can last forever
- Domestic prices are decoupled from PPP at the official exchange rate ($\simeq$ import quota).
- Wedge between the exchange rate and the local price is an implicit growing tax on exports.

Introduce home goods and production to assess effects on resource allocation.

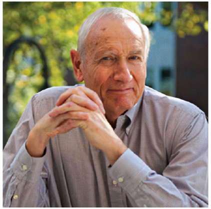
- Inefficient reallocation from the tradeable sector to the production of home goods.
  - The economy converges to international trade autarky, producing only home goods.
- Given the same deficit financing, inflation is
  - lower than under floating exchange rate
  - higher than under fixed exchange rate with free trade
- Consumption falls over time and **real interest rates are below international ones**.

Stand on the shoulders of giants


P. Krugman


G. Calvo

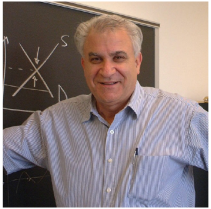

T. Sargent

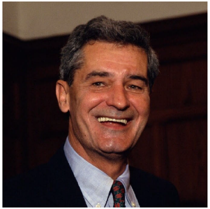

N. Wallace


H. Johnson


J. Frenkel

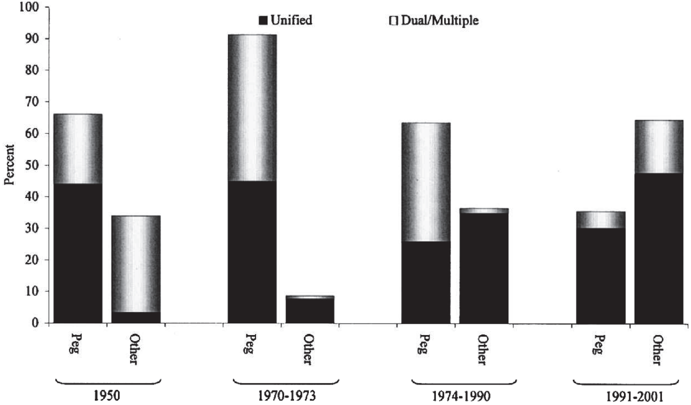

M. Obstfeld


A. Razin


R. E. Lucas

Incidence of Multiple Exchange Rate Arrangements, 1950–2001

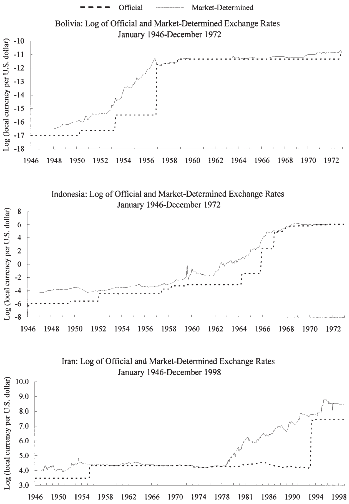
Simplified IMF Classification



Source: Reinhart and Rogoff (2004)

26 countries have multiple exchange rates today

Some Example from Reinhart and Rogoff (2004)



Capital Controls, Inflation, and Growth

Table Inflation and per Capita Real GDP Growth:
A Comparison Of Dual (or Multiple) and Unified Exchange Rate Systems, 1970–2001

Regime	Average annual inflation rate	Average per capita real GDP growth
Unified Exchange Rate	19.8	1.8
Dual (or multiple) exchange rates	162.5	0.8

Source: Reinhart and Rogoff (2004)

Roadmap

1. Modelling capital controls and dual exchange rates
2. Characterization of unsustainable policies and the government's budget constraint
3. A Model of Capital Controls with Free Trade
   * The private sector's budget constraint with anticipated devaluations
   * Optimal behavior with anticipated devaluations
   * Characterization of equilibrium
4. A Model of Capital Controls with Import Restrictions
   * Introduce production and two sectors: T and H
   * Budget constraints and implicit taxes
   * Characterization of equilibrium

Modelling capital controls and dual exchange rates

Start with the balance of payments identity

$$\Delta \text{Reserves}_t + \Delta \text{Private Net Foreign Assets}_t \equiv \text{Current Account}_t$$

Typically, capital controls

- force all current account transactions to go through the central bank (at the official exchange rates), and
- forbid access to foreign exchange from the central bank at the official exchange rate for private foreign asset accumulation

In other words,

$$\Delta \text{Reserves}_t = \text{Current Account}_t$$

$$\boxed{\Delta \text{Private Net Foreign Assets}_t = 0}$$

Private agents can (legally or illegally) exchange foreign assets for domestic currency among themselves at a mutually arranged price.

Shadow exchange rate

Consider a Lucas (1978) tree with a dividend equal to the international interest rate, r.

$$\left.\begin{array}{ll} \text{Offshore price} & 1 \\ \text{Onshore price} & Q_t \end{array}\right\} \text{Ratio} = \text{shadow exchange rate } : Q_t$$

The tree's dividend is a perishable consumption good

The parallel market premium is $q_t \equiv Q_t / E_t$.

It could be thought as the price of the tree in terms of the fruit.

Click for real world example

Onshore interest rates, dual exchange rates, and devaluation

No arbitrage and the domestic return on the foreign currency perpetuity

$$i_t = \frac{rE_t}{Q_t} + \frac{\dot{Q}_t}{Q_t}$$

$$\rho_t \equiv i - \epsilon = \frac{r}{q_t} + \frac{\dot{q}_t}{q_t}$$

Consider the case in which E_t jumps when the regime changes at $t = T$.

Assume $Q(t)$ is continuous in t and $Q_T = E_T^+ \equiv \lim_{\delta \to 0} E_{T+\delta}$.

$$Q_t = E_T^+ e^{-\int_t^T i(s)ds} + r\int_t^T E_s e^{-\int_t^s i(x)dx} ds \qquad \text{discounted } Q + \text{ PV coupons}$$

Unsustainable Monetary and Exchange Rate Policies

The leading case in the literature (Krugman, 1979)

- The rate of growth of central bank credit to the treasury is constant, $\dot{D}_t / D_t = \theta$
- The rate of devaluation is constant, $\epsilon_t = \epsilon < \theta$ for $t < T$

When the central bank's international reserves are zero, at $t = T$, the fixed exchange rate regime ends and the exchange rate floats.

Link with details on the government´s budget constraints

Private Sector's Budget Constraints

The consumer's budget constraints are

$$\frac{\dot{M}_t + \dot{B}_t}{E_t} + q_t \dot{b}_t^* = y - c_t - \tau_t + r b_t^* + \frac{i_t B_t}{E_t} \qquad \text{for } t \neq T$$

$$0 = Q_T \left(b_T^{*+} - b_T^{*-} \right) + \; M_T^+ + B_T^+ - \left(M_T^- + B_T^- \right)$$

Portfolio reallocations at T:

- Under free capital mobility, agents can freely exchange money for foreign currency bonds at the price $Q_T = E_T$.
- Under capital controls $\dot{b}_t^* = b_T^{*+} - b_T^{*-} = 0$ for all t and agents cannot trade away their local currency nominal wealth, $Q_T b_T^{*-} + M_T^- + B_T^-$.

Private Wealth Accumulation and the Intertemporal Budget Constraint

Defining private wealth as $a_t \equiv m_t + b_t + q_t b_t^*$, and recalling $\rho_t \equiv r/q_t + \dot{q}_t/q_t$, we get

$$\dot{a}_t = \rho_t a_t + y - c_t - \tau_t - (\rho_t + \epsilon_t) m_t \qquad \text{for } t \neq T$$

$$a_T^+ - a_T^- = \left(\frac{1}{E_T^+} - \frac{1}{E_T^-}\right)\left(Q_T b_T^{*-} + M_T^- + B_T^-\right) = \left(\frac{E_T^-}{E_T^+} - 1\right) a_T^-$$

If there is an anticipated devaluation:

- wealth is not differentiable at T.
- consumer's have an **anticipated** capital loss on their time T nominal wealth, including $Q_T b_T^*$.

The Consumer's Problem

$$\max_{c_t, m_t} \int_0^\infty u(c_t, m_t) e^{-rt} dt, \text{ subject to } \begin{cases} \dot{a}_t = \rho_t a_t + y - c_t - \tau_t - (\rho_t + \epsilon_t) m_t \text{ for } t \neq T \\ a_T^+ - a_T^- = \left(\frac{E_T^-}{E_T^+} - 1 \right) a_T^- \\ a_0 \text{ given and } \lim_{t \to \infty} a_t e^{-\int_0^t \rho_s ds} \geq 0. \end{cases}$$

Free Capital Mobility

- There cannot be anticipated devaluations and, hence, there are no anticipated jumps in a_t
- Solve a standard problem with $\rho_t = r$
- Krugman (1979)'s case.

Capital Controls

- Consumer's anticipate the capital loss on a_T^- at T.
- Take this into account in the optimal choice of a_T^-.

The Consumer's Optimal Choices

The solution to the household's problem is

$$\frac{u_m(m_t)}{u_c(c_t)} = i_t \qquad \text{for all } t$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{-\frac{u_{cc}(c_t)c_t}{u_c(c_t)}}(\rho_t - r) \qquad \text{for all } t \neq T$$

$$u_c\left(c_T^-\right) = u_c\left(c_T^+\right)\frac{E_T^-}{E_T^+} \qquad \text{Euler equation at } T$$

The intertemporal budget constraint completes the characterization of optimal choices.

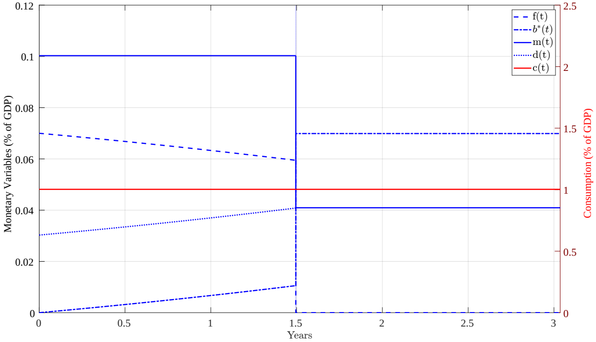
Familiar for $t \neq T$

Consumption is constant for $t \geq T$ as $\rho_t = r$ for $t \geq T$.

For simplicity, assume $u(c,m)$ is homogeneous with $u_{cm} = 0$.

BoP crisis with capital mobility. Krugman (1979)

- Assumptions. link
- Definition of equilibrium. link



Note: $u = \frac{c^{1-\sigma}}{1-\sigma} + \alpha \frac{m^{1-\sigma}}{1-\sigma}$; $\sigma = 2$; $r = 0.04$; $\epsilon = 0$; $\theta = 0.2$; $y = 1$; $f_0 = 0.07$;
$c_{ss} = rf_0 + y$; $m_0 = 0.1c_{ss}$; $d_0 = m_0 - f_0$; $\alpha = 0.1^{\sigma}(r + \epsilon)$

BoP crisis with capital controls

Assumption (Capital Controls)

1. $b_t^* = b_0^*$ *for all* t.
2. m_0, b_0^* *and* f_0 *are a Krugman equilibrium given initial private wealth* a_0^-.

Definition

A *Krugman equilibrium with capital controls* is a switch time, T, allocations $\{c_t, m_t, b_t^*, f_t, \tau_t\}_{t=0}^{\infty}$, shadow exchange rates $\{Q_t\}_{t=0}^{\infty}$, and floating exchange rates $\{E_t\}_{t=T}^{\infty}$ such that given initial conditions $\{D_0, E_0, a_0, a_0^g\}$, interest rates, r, government expenditures and endowments, g, y, the following conditions hold.

1. Nominal interest rates satisfy the no-arbitrage condition.
2. Households optimize given a_0^+ and the sequence of prices r and $\{i_t, E_t, Q_t\}_{t=0}^{\infty}$.
3. T is the smallest t such that $f_T = 0$.
4. The government's intertemporal budget constraint is satisfied.

Equilibrium dynamics with capital controls after T

The equilibrium after T is similar to the one with free capital mobility.

$$
\begin{aligned}
c_t &= c_T^+ = r b_0^* + y - g && \text{for all } t \\
m_t &= m(c_T^+, r+\theta) && \text{for all } t
\end{aligned}
$$

- The final consumption in the first problem, c_T^-, is derived from the Euler condition at T;

$$
\frac{u_c\left(c_T^+\right)}{u_c\left(c_T^-\right)} = \frac{E_T^-}{E_T^+} = \frac{m^+}{m^-} = \frac{m\left(c_T^+, r+\theta\right)}{d_0 e^{(\theta-\epsilon)\mathbf{T}}}.
$$

- **T** is an unknown equilibrium object.
- c_T^- is an increasing function of **T**.

Equilibrium dynamics with capital controls before T

Before T equilibrium dynamics obey

$$\begin{cases} \dot{m}_t = r(m_t + b_0^*) + y - g - c_t + (\theta - \epsilon - r)\, d_0 e^{(\theta-\epsilon)t} & \text{MA BoP} \\ \dot{c}_t = \frac{c_t}{\sigma}\left(\underbrace{\overbrace{\frac{u_m(c_t,m_t)}{u_c(c_t,m_t)}}^{i_t} - \epsilon}_{\rho_t} - r\right) & \text{Optimal behavior} \\ \dot{f}_t = r f_t + y - g - c_t & \text{CA identity} \end{cases}$$

- The monetary approach to the balance of payments (Frenkel and Johnson (1976))

$$\dot{m}_t - \dot{d}_t = \dot{f}_t = r(f_t + b_t^*) + y - g - c_t$$

- c and m are independent of f.
- For initial conditions $\{c_0, m_0, f_0\}$ the dynamic system returns $\{c_T^-, m_T^-, f_T^-\} = \{c_T^-, d_T, 0\}$

Equilibrium boundary conditions before T

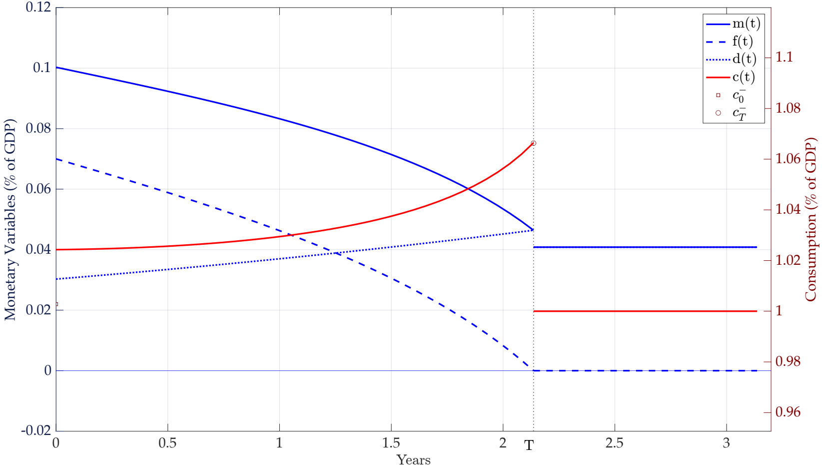
In order to characterize the equilibrium, we need boundary conditions for the system in $\{m_t, c_t, f_t\}$ between 0 and T.

$$\begin{cases} m_0 = m(r+\epsilon)\left[r\left(f_0+b_0\right)+y-g\right] & \text{Initial } m \\ f_0 = m_0 - d_0 & f_0 \text{ is a Krugman equilibrium} \\ u_c\left(c_T^-\right) = u_c\left(c_T^+\right)\frac{d_0 e^{(\theta-\epsilon)\mathbf{T}}}{m\left(c_T^+, r+\theta\right)} & \text{Euler condition at T} \\ f_{\mathbf{T}} = 0 & \text{Switch time and final } f_T \end{cases}$$

Shooting algorithm

Find $\{c_0, T\}$ such that $\begin{cases} f(c_0, T) = 0 \Rightarrow T = T(c_0) & \text{terminal condition for } f \\ c(c_0, T(c_0)) = c^-_{T(c_0)} & \text{terminal condition for } c \end{cases}$

BoP crisis with capital controls: allocations



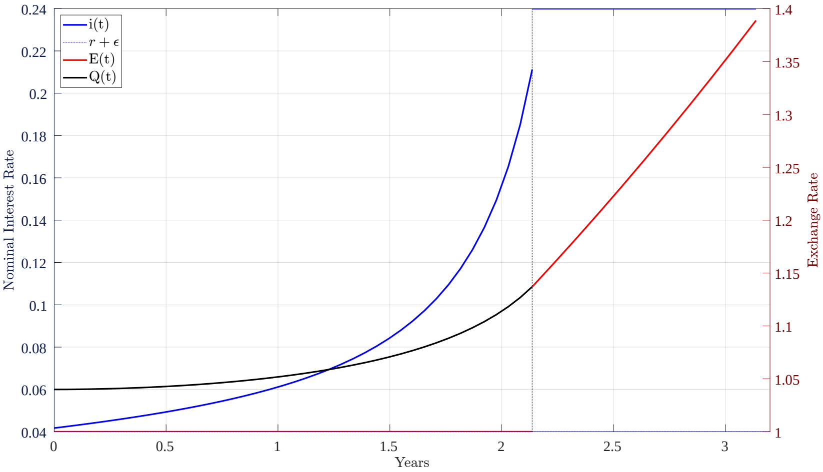
Before T

- Consumption booms
- Money demand falls to avoid capital loss.
- Households "buy" reserves through consumption.

Jumps at T

- Consumption tanks (contractionary devaluation?)
- Money demand falls (higher interest rate).

BoP crisis with capital controls: prices



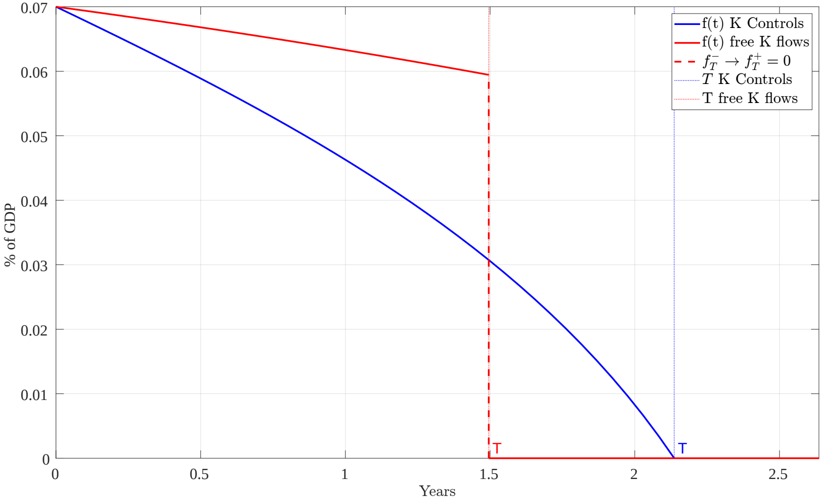
Before T

- Inflation is $\pi = 0$
- Q_t jumps at $t = 0$ and converges to E_T^+.
- Interest rates rise, consistent with fall in $\frac{m}{c}$.

Jumps at T

- Inflation jumps to $\pi = \theta$
- Prices jump $\Rightarrow$ real value of wealth falls
- interest rates jump

BoP crisis with and without capital controls: reserve dynamics



- Capital controls are successful at delaying the BoP crisis.
- The loss of reserves under capital controls is gradual, but fast as money demand is falling.

Delayed monetization

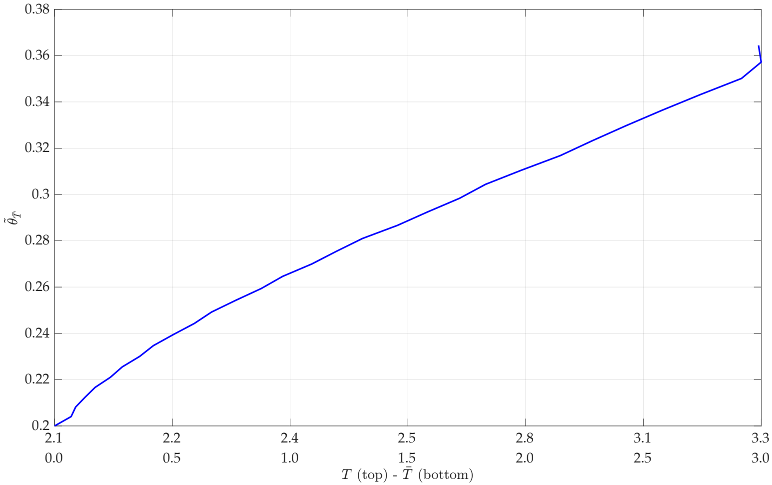
- A Wallace (1981) neutrality theorem holds in the Krugman model with free capital mobility.
- A Sargent and Wallace (1981) delayed monetization of deficits extends the life of the peg at the cost of higher inflation later.
  - Finance same deficit with a lower money supply at T: $\frac{\text{Deficit}}{M_T}$ is larger.
  - Deficit is larger due to debt service

$$B_{\bar{T}} = \int_0^{\bar{T}} \underbrace{\theta D_0 e^{\theta t}}_{\text{Deficit}_t} \underbrace{e^{\int_t^{\bar{T}} i_s ds}}_{\text{Interest}} dt$$

$$\tilde{\theta}_t = \begin{cases} 0 & \text{for } t \leq \bar{T} \\ \theta e^{\theta \bar{T}} + \rho_t \frac{B_{\bar{T}}}{D_0} e^{-\theta(t-\bar{T})} & \text{for } \bar{T} < t \end{cases}$$

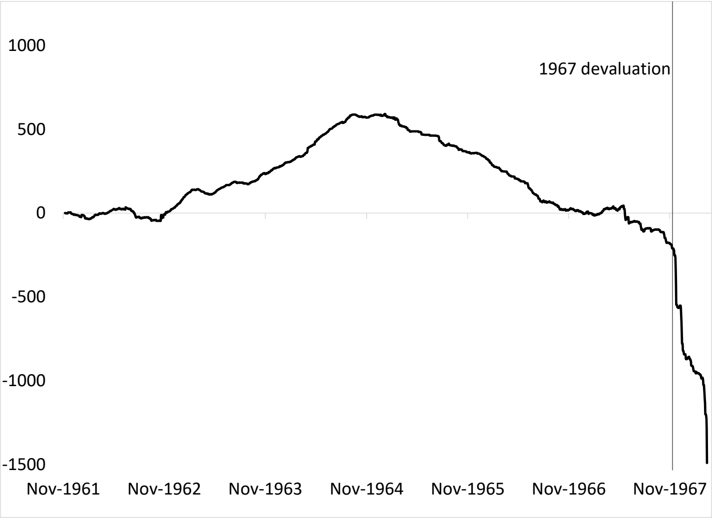
Delayed monetization frontier

Regime Switch delay v. higher Inflation later

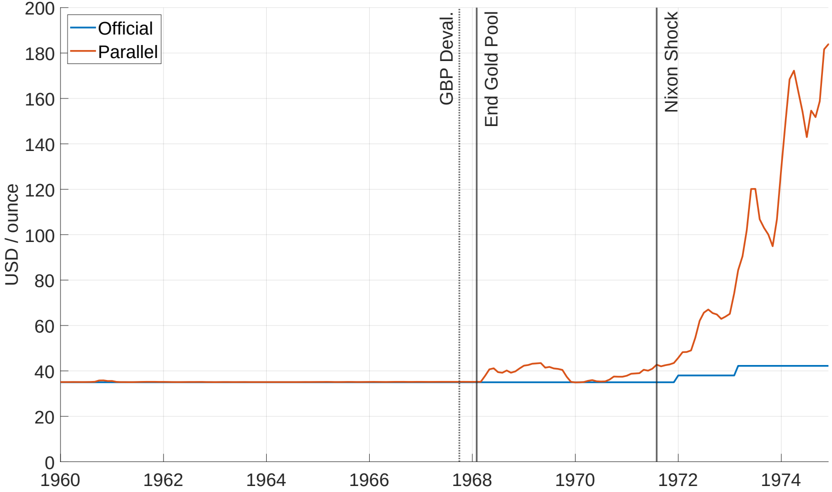


Does the model apply to the demise of the Woods exchange rate regime?

Cumulative Gold Pool Intervention 1961-1968

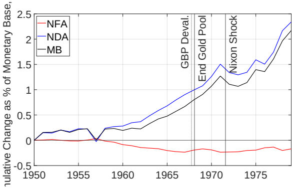


The price of gold

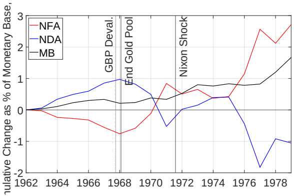


The Gold Pool's Inconsistencies: Central Bank Balance Sheets

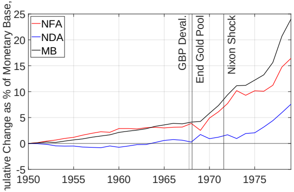
(a) USA



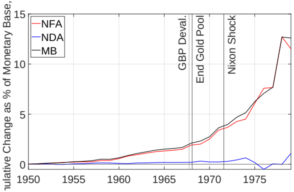
(b) Great Britain



(c) Germany



(d) Switzerland



Source: IFS.

The Adjustment Problem:

Real Exchange Rate Appreciation and Current Account Deficit

Consider the economy with capital controls and $\theta > \epsilon$

Add (trivially) a non-traded good

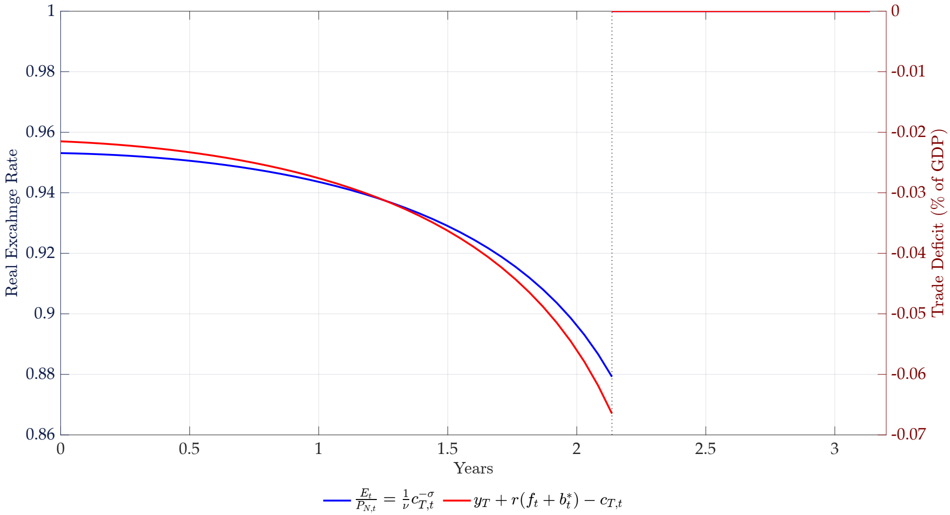
- Preferences $u(c_{T,t}) + \nu u(c_{N,t}) + \alpha u(m_t)$
- Endowment of N good is $y_N = 1$
- Additional equilibrium condition

$$\frac{E_t}{P_{N,t}} = \frac{u'(c_{T,t})}{\nu u'(c_{N,t})} = \frac{1}{\nu} c_{T,t}^{-\sigma}$$

It appears there is an adjustment problem

The Adjustment Problem?

Real Exchange Rate Appreciation and Current Account Deficit



How to avoid a BoP crisis: Hello Import Restrictions!

- Capital controls may delay, but cannot avoid a BoP crisis.
- When the crisis hits, it is painful.
  - Consumption falls
  - A jump in the price level imposes a capital loss on private agents
  - Inflation jumps
- Avoid devaluation by imposing import restrictions that preclude reserves from falling.
- We'll model these policies and their costs.
  - Introduce production and a labor market
  - Two sector: home goods and traded goods
  - Efficient allocation is the endowment economy of the previous models.

Import restrictions

Set import restrictions so that reserves are constant: $\dot{f}_t = 0$

Assumption (Import restriction)

$$c_{T,t} \leq \bar{c}_t \textit{ for all } t$$

Recall the current account dynamics: $\dot{f}_t = r\left(f_t + b_t^*\right) + y_{Tt}\left(n_{Tt}\right) - g - c_{Tt}$

Import restrictions act like trade quotas: domestic prices are delinked from import parity prices.

Non-traded goods and resource allocation

Assumption (Perfect substitutability between H and T goods)

Assume the goods, H and T, are perfect substitutes in consumption;

$$u\left(c_{H,t}, c_{T,t}, m_t\right) = u\left(c_t, m_t\right) \text{ where } c_t = c_{H,t} + c_{T,t}.$$

Assumption (Production technology)

The production functions for H and T goods are;

i $y_{T,t} = y\left(\ell_{T,t}\right)$ *with* $y' > 0$, $y'' < 0$, *and* $y'(1) = 1$

ii $y_{H,t} = \ell_{H,t}$

iii $\ell_{H,t} + \ell_{T,t} = 1$

In an efficient allocation, only traded goods are produced and consumed; $\ell_{T,t} = 1$, $y_{H,t} = \ell_{H,t} = 0$, $y_{T,t} = y\left(1\right)$ and the relative price between the two goods is one.

Price wedges, taxes, and budget constraints

The private and government budget constraints are

$$\frac{\dot{M}_t + \dot{B}_t}{P_t} + q_t \dot{b}_t^* = rb^* + i_t \frac{B_t}{P_t} + \underbrace{\frac{E_t}{P_t}}_{1-\text{ implicit tax}} y\left(n_{T,t}\right) - c_{T,t} - \tau_t + \frac{p_{Ht}}{P_t}\left(1 - n_{T,t} - c_{H,t}\right)$$

$$\dot{f}_t - \frac{\dot{M}_t + \dot{B}_t}{P_t} = rf_t - i_t \frac{B_t}{P_t} + \tau_t - g + \underbrace{\left(1 - \frac{E_t}{P_t}\right)}_{\text{implicit tax}} y\left(n_{T,t}\right).$$

All tradable goods are exported at the official exchange rate

The government uses export proceeds to buy the consumption good, which it resells at price P_t.

Profits from importing are transferred to consumers.

Simplifies introducing an agent that profits from access to imports at price E_t.

The Household's Problem

The consumer's problem is to choose $\{c_{H,t}, c_{T,t}, n_{H,t}, n_{T,t}, m_t\}$ to solve the problem

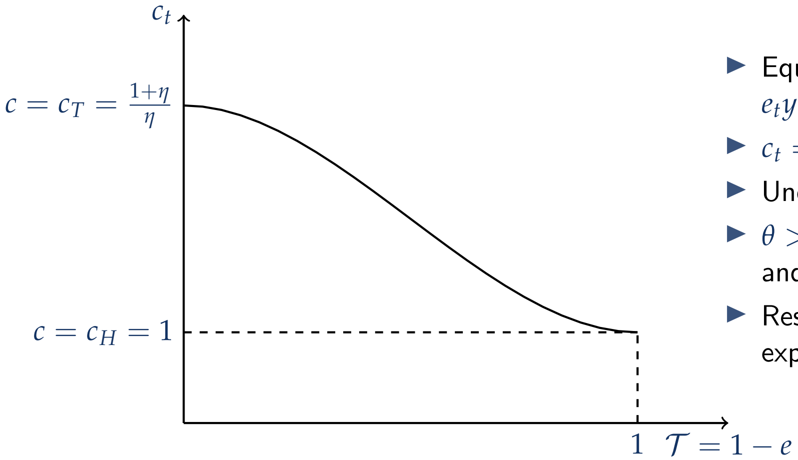
$$\max_{c_{H,t},c_{T,t},n_{T,t},m_t} \int_0^\infty u\left(c_{H,t} + c_{T,t}, m_t\right) e^{-rt} dt \quad \text{subject to}$$

$$\begin{cases} \dot{a}_t = \rho_t a_t + \frac{E_t}{P_t} y(n_{Tt}) - c_{T,t} - \tau_t - i_t m_t + \frac{p_{Ht}}{P_t}\left(1 - n_{T,t} - c_{H,t}\right) \\ a_0 \text{ given} \end{cases}$$

Find prices at which the household's optimal choice satisfies the import constraint

$$c_{Tt} = \underbrace{\bar{c}_t = r(b_0^* + f_0) + y(n_{Tt}) - g}_{\bar{c}_t : \dot{f} = 0}$$

Import Controls, Inflation and Resource Misallocation. Details



- Equilibrium allocation: $e_t y'(n_T) = 1 =$ Mg Prod n_H
- $c_t = y'(n_{Tt}) + 1 - n_{Tt}$
- Under free trade $e = 1$
- $\theta > \epsilon$ with import controls $\Rightarrow \dot{e} < 0$ and $e_t \to 0$ as $t \to \infty$.
- Resources misallocation inefficiently expands the home good sector

Equilibrium Dynamics: Money, Exchange Rate, and Inflation

The money market equilibrium, $M_t/P_t = m_t(i_t, c_t)$ implies

$$e_t = \frac{E_t}{P_t} = \frac{E_t m_t}{M_t} = \frac{\ell(i_t)c(e_t)}{f_0 + d_0 e^{(\theta-\epsilon)t}},$$

which, in equilibrium, is

$$\begin{cases} e_t = \dfrac{\ell\left(r+\epsilon+\left(\frac{\sigma(1-e_t)n'(e_t)}{c(e_t)}-1\right)\frac{\dot{e}_t}{e_t}\right)c(e_t)}{f_0+d_0e^{(\theta-\epsilon)t}} \\ \\ \lim_{t\to\infty} e_t = 0 \end{cases}$$

- Inflation is contained in the interval $\epsilon < \pi_t \leq \theta$
- The limiting inflation is θ, $\lim\limits_{t\to\infty} \pi_t = \theta$.
- The wedge $e = \frac{E}{P}$ falls over time, $\dot{e}_t < 0$

Some Prices of Interest

- Price of Lucas tree in terms of its fruit

$$\frac{Q_t}{P_t} = q_t = r \int_t^{\infty} e^{-\int_t^s \rho_u du} ds = \int_t^{\infty} \frac{e^{-r(s-t)}}{\int_t^{\infty} e^{-r(s-t)}} e^{\int_t^s (r-\rho_u) du} ds$$

- Parallel market premium

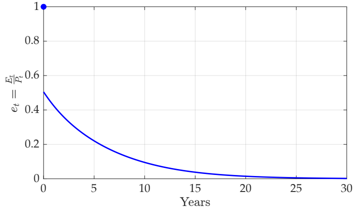
$$\frac{Q_t}{E_t} = \frac{q_t}{e_t}$$

- Price level change of unanticipated exchange rate unification

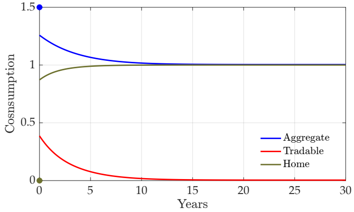
$$\frac{P_t^{\theta}}{P_t} = \frac{\ell(\rho_t + \pi_t) c(e_t)}{\ell(r+\theta) \ (r(f+b^*) + y_T(1))}$$

- Properties
  - $\dot{C} < 0 \Rightarrow \rho_t < r$ and as consumption converges to $c(e=0))$, $\rho_t \to r$
  - $q_t \geq 1$ and $\lim_{t\to\infty} q_t = 1$.
  - $\frac{Q_t}{E_t} > 1$ and $\lim_{t\to\infty} \frac{Q_t}{E_t} = \infty$.

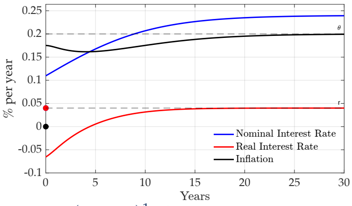
(a) Real Exchange Rate $e = \frac{E}{P}$



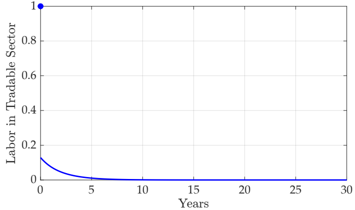
(b) Consumption



(c) Interest Rates and Inflation

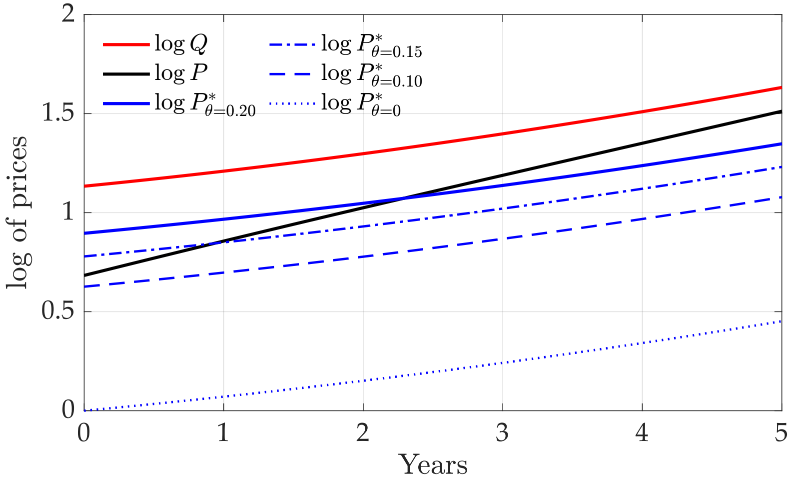


(d) Labor in T sector



Note: $u = \frac{(c_T + c_H)^{1-\sigma}}{1-\sigma} + \alpha \frac{m^{1-\sigma}}{1-\sigma}$; $\sigma = 2$; $r = 0.04$; $\epsilon = 0$; $\theta = 0.2$; $y = 1$; $f_0 = 0.07$; $c_{ss} = rf_0 + y$; $m_0 = 0.1c_{ss}$; $d_0 = m_0 - f_0$; $\alpha = 0.1^{\sigma}(r + \epsilon)$; $y_T = \frac{1+\eta}{\eta} l_T^{\frac{\eta}{1+\eta}}$; $y_N = 1 - l_T$; $\eta = 2$.

Exchange Rate Unification and Trade Liberalization: Prices



Recall $\rho_t + \pi_t < r + \theta$ and $c(e) < c(1)$

$$\frac{Q_t}{P_t} = r \int_t^\infty e^{-\int_t^s \rho_u du} ds$$

$$\frac{P_t^\theta}{P_t} = \frac{\ell(\rho_t + \pi_t)c(e_t)}{\ell(r+\theta)c(1)}$$

Perils of an Unanticipated Liberalization

- Consider an economy with $\theta > \epsilon$, capital controls and import restrictions
- Liberalization without fiscal reform induces a jump in the price level due to a fall in the demand for money when the interest rate rises from $\rho_t + \pi_t$ to $r + \theta$ and consumption is not too low. .
- Real interest rates after liberalization increase from ρ_t to r, which will affect the cost of servicing domestically placed debt.

Future steps

- Investigate the effect of delayed monetization with import controls and low interest rates.
- Investigate equilibrium with an anticipated liberalization at a known future date.

Final Remarks

We have shown that in economies with inconsistent monetary and fiscal policies that lead to balance of payments crisis (Krugman, 1979).:

- Capital controls can delay the crises at the cost of
  - an anticipated devaluation that imposes a capital loss on consumers, and
  - a consumption boom-bust cycle

  Does the model apply to the end of Bretton Woods?

- Import controls can delay the crisis forever at the cost of severe resource misallocation that shuts down the export sector, leading to international trade autarky.
  - Capital control liberalization raises interest rates
  - This poses a challenge
    - Price level after liberalization
    - Debt service
    - More research is needed

ADDITIONAL MATERIAL

The Government's budget constraint

The consolidated public sector's budget constraint is

$$\dot{f}_t = (\tau_t - g) + r f_t - i_t \frac{B_t^g}{E_t} + \frac{\dot{M}_t + \dot{B}_t^g}{E_t} \qquad \text{for } t \neq T$$

$$Q_T \left(f_T^+ - f_T^-\right) = M_T^+ + B_T^{+g} - M_T^- - B_T^{-g} \qquad \text{for } T,$$

- Under free capital mobility, at T there may be a portfolio reallocation at the price $Q_T = E_T$.
- Under capital controls, $\left(f_T^+ - f_T^-\right) = 0$ and, plausibly, $Q_T \neq E_T$.

Back

Separating the treasury's accounts from the central bank's

The central bank's balance sheet is

$$E_t f_t + D_t = M_t,$$

where D_t denotes domestic credit.

It is convenient to denominate the CB's balance sheet in foreign currency,

$$f_t + d_t = m_t, \tag{1}$$

where $d_t \equiv D_t / E_t$ and $m_t \equiv M_t / E_t$.

Unfunded government deficits are financed with central bank credit;

$$\frac{\dot{D}_t}{E_t} = i_t \frac{B_t^g}{E_t} + g - \tau_t - (r + \epsilon_t) f_t - \frac{\dot{B}_t^g}{E_t}. \tag{2}$$

Back

Krugman (1979) Equilibrium

Assumption (Monetary and fiscal policy)

1. *Monetary and exchange rate policy*
   1.1 *The rate of growth of domestic credit,* $\dot{D}_t / D_t = \theta$*,*
   1.2 *The rate of crawl of the "fixed" exchange rate,* ϵ_t*, is constant for* $t \leq T$*.*
   1.3 *The fixed exchange rate regime is unsustainable:* $\theta > \epsilon$
2. *Fiscal dominance*
   2.1 *Government's deficit is financed with central bank credit,* $\dot{D}_t = [g - \tau_t - (r + \epsilon_t) f_t] E_t$
   2.2 *No access to credit,* $\forall t : B_t = 0$*.*
3. *Regime change: at time* T *when reserves are zero the exchange rate regime switches to a floating one.*

Assumption (Free capital mobility)

Free capital mobility and positive interest rates $\rho_t = r > 0$*.*

Back

Definition of Equilibrium with Free Capital Mobility

Definition (Krugman Equilibrium)

A *Krugman equilibrium with free capital mobility* is a regime switch time, T, a sequence of allocations $\{c_t, m_t, b_t^*, f_t, \tau_t\}_{t=0}^{\infty}$, and floating exchange rates $\{E_t\}_{t=T}^{\infty}$ such that given initial conditions $\{D_0, E_0, a_0, a_0^g\}$, international interest rates, r, assumption 5, assumption 6, and a sequence of government expenditures g the following conditions hold.

1. Nominal interest rates satisfy the no-arbitrage condition $i = r + \epsilon$
2. Households optimize given a_0 and the sequence of prices r and $\{i_t\}_{t=0}^{\infty}$.
3. T is the smallest t such that $f_T^+ = 0$.
4. The government's intertemporal budget constraint is satisfied.

Back

Characterization of Static Equilibrium.

Let $e_t \equiv \frac{E_t}{P_t}$.

$$
\begin{aligned}
p_{Ht} =& P_t && \text{(Interior solution)}\\
1 =& e_t y'(n_{T,t}) \iff n_{Tt} = n(e_t), \quad n'(e_t) > 0 && \text{(Optimal allocation)}\\
c_{T,t} =& r(f_0 + b_0^*) + y(n(e_t)) - g && \text{(Import constraint)}\\
c_{Ht} =& 1 - n(e_t) && \text{(Aggregate Consistency)}\\
c_t =& 1 - n(e_t) + r(f_0 + b_0^*) + y(n(e_t)) - g && \text{(Consumption Aggregator)}
\end{aligned}
$$

- The real exchange rate e determines the labor allocation, production and consumption.
- The dynamics of e are derived from the money market equilibrium
  - $\theta > \epsilon$ implies that e decreases over time

Back

Properties of equilibria as a function of e

1. If the import constraint does not bind, the economy is the same as in previous models.
2. If the wedge is one, $e \to 0$, the economy goes to autarky.

$$e = 1 \Rightarrow \begin{cases} n_T = n(1) = 1 \\ y_H = n_H = c_H = 0 \\ c = c_T = r\left(f_0 + b_0^*\right) + y\left(1\right) - g \end{cases}$$

$$e \to 0 \Rightarrow \begin{cases} n_T = y_T(n_{Tt}) = 0 \\ y_H = n_H = c_H = 1 \\ c_T = r\left(f_0 + b_0^*\right) - g \\ c = 1 + r\left(f_0 + b_0^*\right) - g \end{cases}$$

Back

Money Demand, Savings Decisions and Interest Rates

$$\frac{u_m(m_t)}{u_c(c_t)} = i_t = \rho_t + \pi_t \qquad \text{(Money Demand)}$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(\rho_t - r) \qquad \text{(Optimal savings - Euler equation)}$$

$$\dot{c}_t = n'(e)\left(\frac{1}{e} - 1\right)\dot{e}_t$$

We can write the money demand as

$$m_t = \ell(i_t)c_t = \ell(r + \epsilon + \sigma\frac{\dot{c}_t}{c_t} + \pi - \epsilon)c_t(e_t)$$

Back

Equilibrium Dynamics: Money, Exchange Rate, and Inflation

The money market equilibrium, $M_t/p_t = m_t(i_t, c_t)$ implies

$$e_t = \frac{E_t}{P_t} = \frac{E_t m_t}{M_t} = \frac{\ell(i_t)c(e_t)}{f_0 + d_0 e^{(\theta-\epsilon)t}},$$

implies

$$e_t = \frac{\ell\left(r + \epsilon + \left(\frac{\sigma(1-e_t)n'(e_t)}{c(e_t)} - 1\right)\frac{\dot{e}_t}{e_t}\right)c(e_t)}{f_0 + d_0 e^{(\theta-\epsilon)t}}$$

$$\lim_{t\to\infty} e_t = 0$$

- Inflation is contained in the interval $\epsilon < \pi_t \leq \theta$
- The limiting inflation is θ, $\lim_{t\to\infty} \pi_t = \theta$.
- The wedge $e = \frac{E}{P}$ falls over time, $\dot{e}_t < 0$
- The limiting wedge is $\lim_{t\to\infty} e_t = 0$.

Back

Example

- Assume $b_0 = f_0 = g = 0$
- Consider the technology: $y_T = \frac{1+\eta}{\eta} n_T^{\frac{\eta}{1+\eta}}$ with $\eta = 2$.
- Consumers supply of T good: $n_T(e_t) = e_t^{1+\eta} \Rightarrow y_T(e_t) = \frac{1+\eta}{\eta} e_t^{\eta}$.
- Consumption: $c = \underbrace{\frac{1+\eta}{\eta} e_t^{\eta}}_{c_T} + \underbrace{1 - e_t^{1+\eta}}_{c_H}$
- Recall $\mathcal{T} = 1 - e$

Back

Frenkel, Jacob A. and Harry G. Johnson. 1976. *The Monetary Approach to the Balance of Payments*. Toronto and Buffalo: University of Toronto Press.

Frenkel, Jacob A. and Assaf Razin. 1989. "Exchange-rate management viewed as tax policies." *European Economic Review* 33 (4):761–781.

Krugman, Paul. 1979. "A Model of Balance-of-Payments Crises." *Journal of Money, Credit and Banking* 11 (3):311–325. URL http://www.jstor.org/stable/1991793.

Lucas, Robert E. 1978. "Asset Prices in an Exchange Economy." *Econometrica* 46 (6):1429–1445.

Park, Daekuen and Jeffrey Sachs. 1987. "Capital Controls and the Timing of Exchange Regime Collapse." Working Paper 2250, National Bureau of Economic Research. URL http://www.nber.org/papers/w2250.

Reinhart, Carmen M. and Kenneth S. Rogoff. 2004. "The Modern History of Exchange Rate Arrangements: A Reinterpretation." *The Quarterly Journal of Economics* 119 (1):1–48.

Sargent, Thomas J. and Neil Wallace. 1973. "The Stability of Models of Money and Growth with Perfect Foresight." *Econometrica* 41 (6):1043–1048. URL http://www.jstor.org/stable/1914034.

———. 1981. "Some unpleasant monetarist arithmetic." *Quarterly Review* 5 (Fall):1–17.

Wallace, Neil. 1981. “A Modigliani-Miller Theorem for Open-Market Operations.” *The American Economic Review* 71 (3):267–274.